Radar Systems Engineering

Lecture 13

Clutter Rejection

Part 2 - Doppler Filtering

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Guest Lecturer

Block Diagram of Radar System

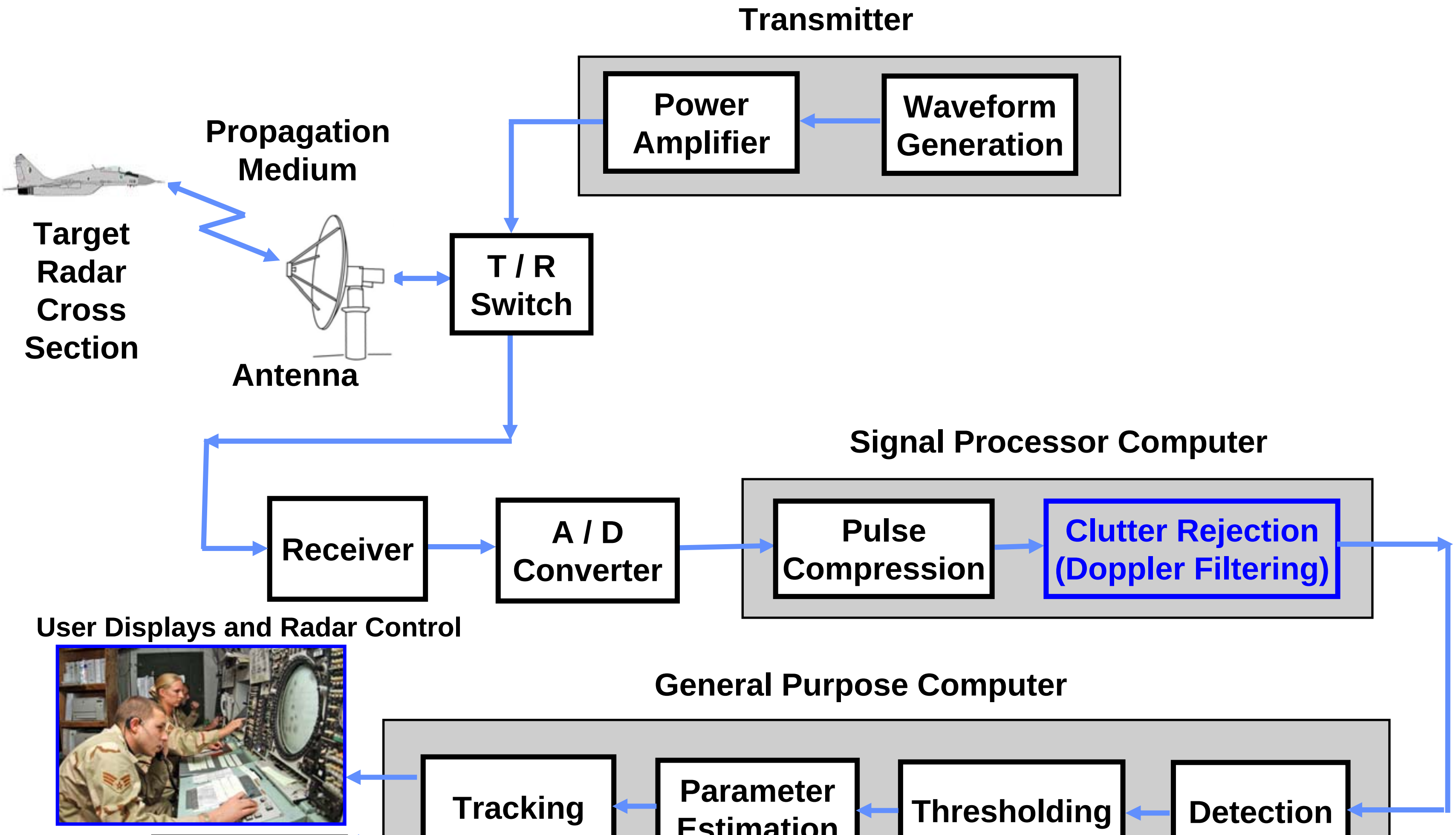


Photo Image
Courtesy of US Air Force

Outline

- Introduction

- Problem perspective
  - Burst Waveforms and their properties
  - The impact of Moore's Law on radar Signal Processing
    - Past, present, and the future

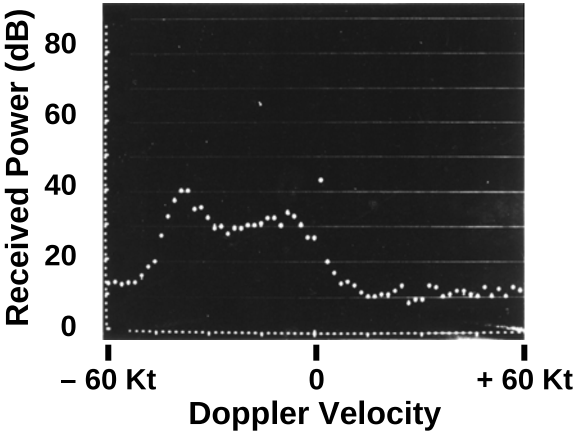
- Pulse Doppler Processing Techniques
  - Description of pulse Doppler processing
  - Low PRF Example – Moving Target Detector (MTD)
  - Range and Doppler Ambiguities
  - Ambiguity Resolution - Chinese remainder theorem
  - The "Ambiguity Function"
  - Preview of Airborne Pulse Doppler issues

- Summary

The Problem- Not Just Ground Clutter

- **Ground Clutter**
  - Can be intense and discrete
  - Can be 50 to 60 dB > than target
  - Doppler velocity zero for ground based radars
    - Doppler spread small
- **Rain Clutter**
  - Diffuse and windblown
  - Can be 30 + dB > than target
  - Doppler zero for ground based radars
    - Doppler spread small

Doppler Spectrum of Rain



Courtesy of FAA

- **Sea Clutter**
  - Less intense than ground
    - By 20 to 30 dB
    - Often more diffuse
  - Doppler velocity varies for based radars (ship speed & wind speed)
    - Doppler spread moderate
- **Bird Clutter**
  - 100s to 10,000s to point targets
  - Doppler velocity - 0 to 60 knots
    - Doppler of single bird has little change
    - Flocks of birds can fill 0 to 60 knots of Doppler space
    - Big issue for very small targets

A one filter with a notch at zero

Doppler will not adequately reject rain

Issues with MTI Cancellers

- Typically they process a few (2-5) pulses at a time, so it is near to impossible to shape them as well as you could if filter had an input of 8-10 pulses
  - 2 pulse MTI canceller is very broad in Doppler space

- A set of pass band Doppler filters, using 8-10 pulses) can be constructed having:
    - A notch at zero Doppler to reject ground clutter
    - A set of passband filters that can detect targets where no rain is present

- Before the mid 1970s, the technology, to cost effectively implement pulse Doppler solutions to the simultaneous ground and rain clutter was not available

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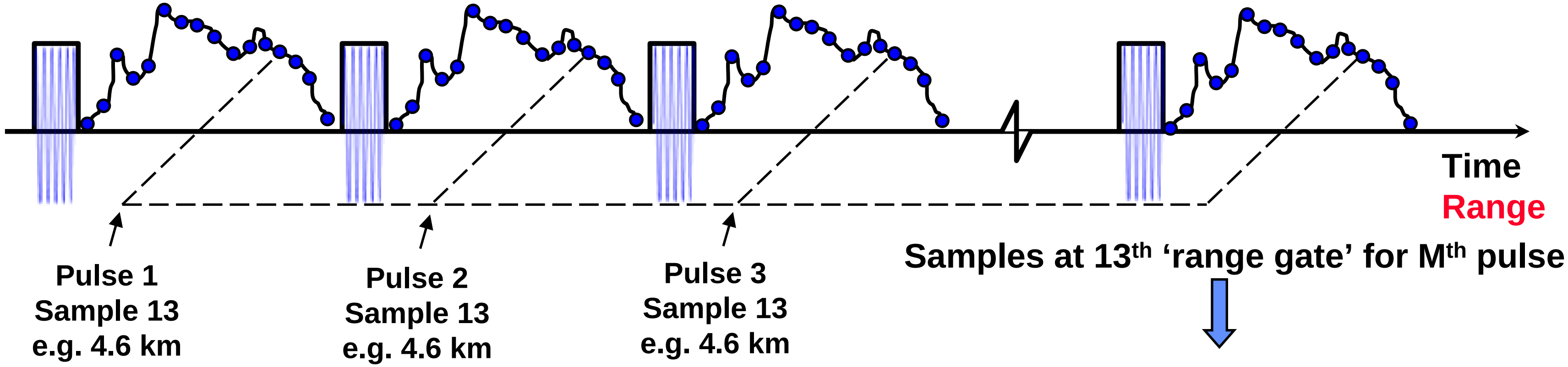
Utility of Burst Waveforms for Clutter Rejection



- **A burst waveform offers a method of collecting M sequential samples an each range - CPI cell.**
- **These samples can be linearly processed through a set of pass band filters that will**
  - **Detect targets within a range of Doppler velocities and simultaneously reject clutter that is in their low sidelobes**
  - **If the pass band filters are narrow enough in frequency, a measurement of the Doppler velocity of the target that passes through them can be made**

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Impact of Moore’s Law of Radar Processing

- Tremendous advances in A/D Converter technology
- In the 1970s, a 10 bit 5 MHz A//D was near the commercial state of the art
  - 30 lbs and 3” of rack space
  - Now it is not only on a chip, but many more bits and much higher sample rates are available
- For a 60 nmi aircraft surveillance radar, with a mechanically scanning antenna, the new computational processing advances allowed the number of range-azimuth-Doppler cells being individually thresholded from a several thousand to several million per radar scan
  - These advances allowed aircraft to be reliably detected in rain
  - Much better detection of aircraft in ground clutter
  - Low false alarm rates that allowed the radar and beacon sensor systems to be seamlessly integrated
- In the future, expect that advances in processing technology will allow, implementation of new techniques, which today are seemingly impossible to implement

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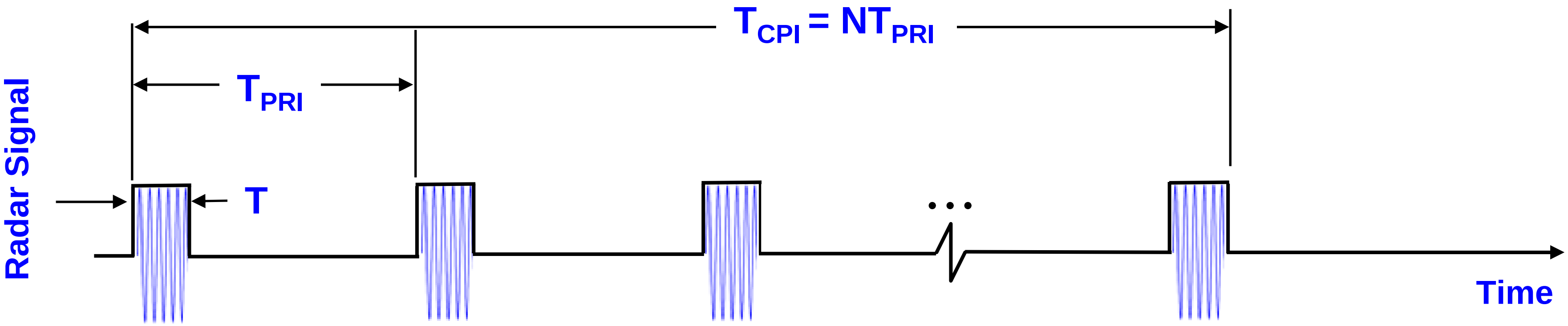
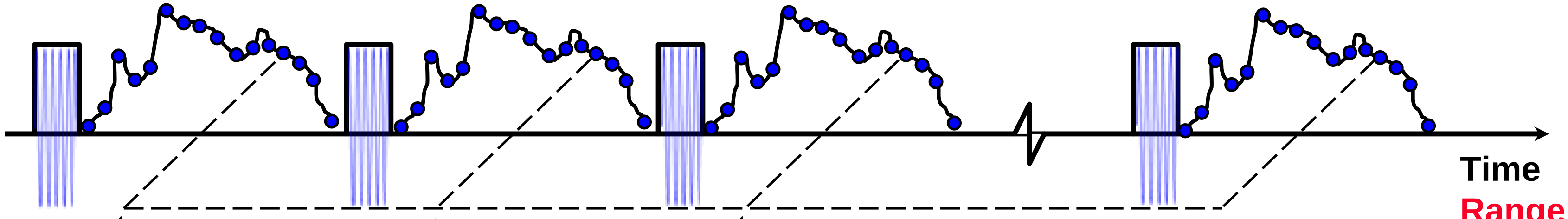
Waveforms for Pulse Doppler Processing Revisited



For Airport Surveillance Radar

T	$=$	Pulse Length	1 μsec
B	$= 1/T$	Bandwidth	1 MHz
T_{PRI}	$=$	Pulse Repetition Interval (PRI)	1 msec
f_P	$= 1/T_{PRI}$	Pulse Repetition Frequency (PRF)	1 KHz
δ	$= T / T_{PRI}$	Duty Cycle (%)	.1 %
T_{CPI}	$= NT_{PRI}$	Coherent Processing Interval (CPI)	10 pulses
N	$=$	Number of pulses in the CPI N = 2, 3, or 4 for MTI N usually much greater (8 to ~1000) for Pulse Doppler	

Data Collection for Doppler Processing

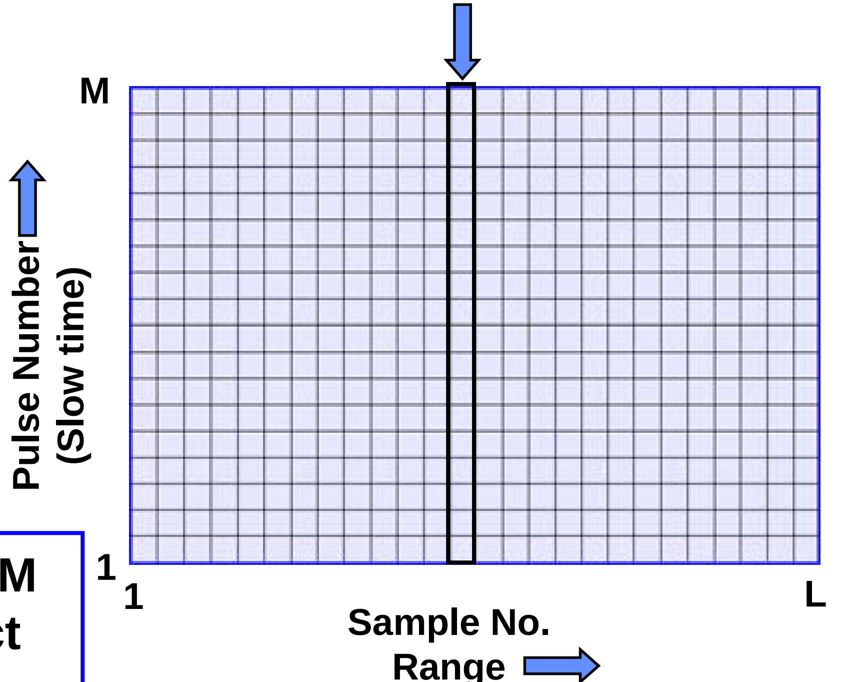


Pulse 1
Sample 13
e.g. 4.6 km

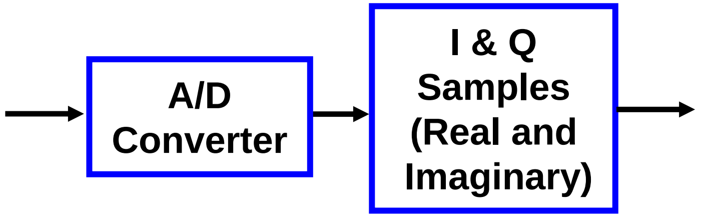
Pulse 2
Sample 13
e.g. 4.6 km

Pulse 3
Sample 13
e.g. 4.6 km

Samples at 13th ‘range gate’ for Mth pulse



Sample No.
Range



A/D Converter

I & Q Samples (Real and Imaginary)

Complex I / Q samples

(the complex envelope of received waveform)

What is the optimum way to process the M voltage samples in a range gate to detect the target and reject clutter?

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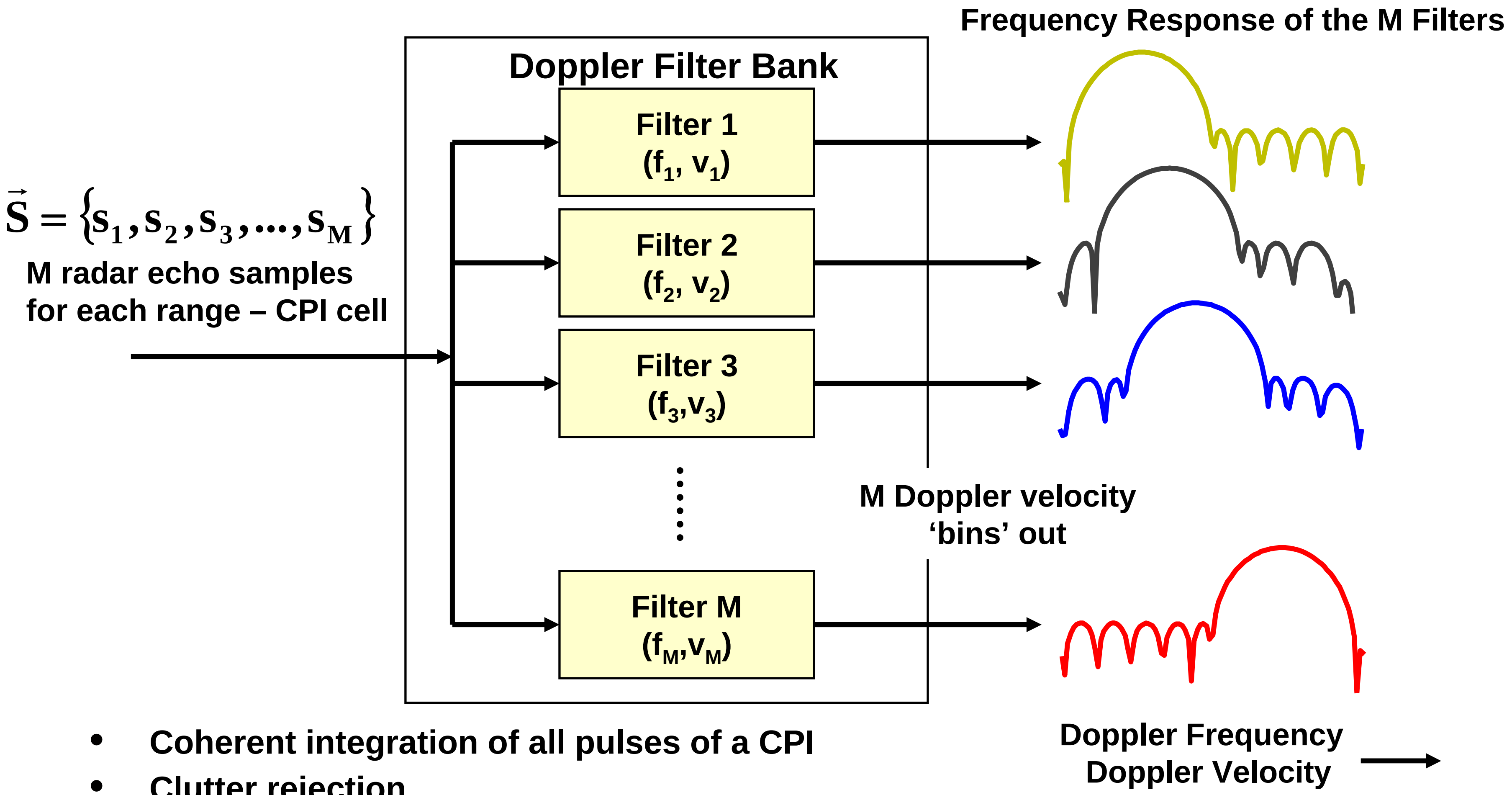
Pulse Doppler Processing - Cartoon



$$\vec{S} = \{s_1, s_2, s_3, \ldots, s_M\}$$

M radar echo samples for each range – CPI cell

- Coherent integration of all pulses of a CPI
- Clutter rejection
- Resolving targets into different velocity segments and allowing for fine-grain target radial velocity estimation

Viewgraph Courtesy of MIT Lincoln Laboratory
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Optimization of Pulse Doppler Processing

- MTI Improvement Factor, I , already introduced in previous lecture is

$$I\,(f_d) = \left. \frac{(\text{Signal / Clutter})_{out}}{(\text{Signal / Clutter})_{in}} \right|$$

- The next question “In the presence of “colored noise” (ground clutter, rain & noise), what are the weights, $W_i(f_D)$, by which the M input signal (+ clutter) samples, S_i , should by multiplied by so that the S/(C+N) will be maximized?
  - Note that the optimum set on weights depends on f_D
  - Also on the number of pulses processed, M

- In the late 1960s, the solution was developed by 2 independent sets of researchers (See Reference14 and 15)

Optimum MTI Improvement Factor

- **Problem**
  - What is the optimum way (maximize $S/(N+C+I)$ to linearly process M complex radar echoes, V_i , in the presence of noise, clutter returns (ground, rain, sea, etc.) and interference?
- **Answer:**

$$R = \left| \sum_{i=1}^{M} W_i V_i \right|^2$$

$$\boxed{I_{OPT} = \sum_i \overline{S}_k \, W_k^{OPT}}$$

  - where $W_i^{OPT} = k \sum_{j=1}^{M} M_{ij}^{-1} S_j$

V_i = Sampled voltage (sum of target echo, clutter, noise, etc.)

M_{ij} = Covariance matrix of clutter, noise, etc

S_i = Signal vector

k = arbitrary constant

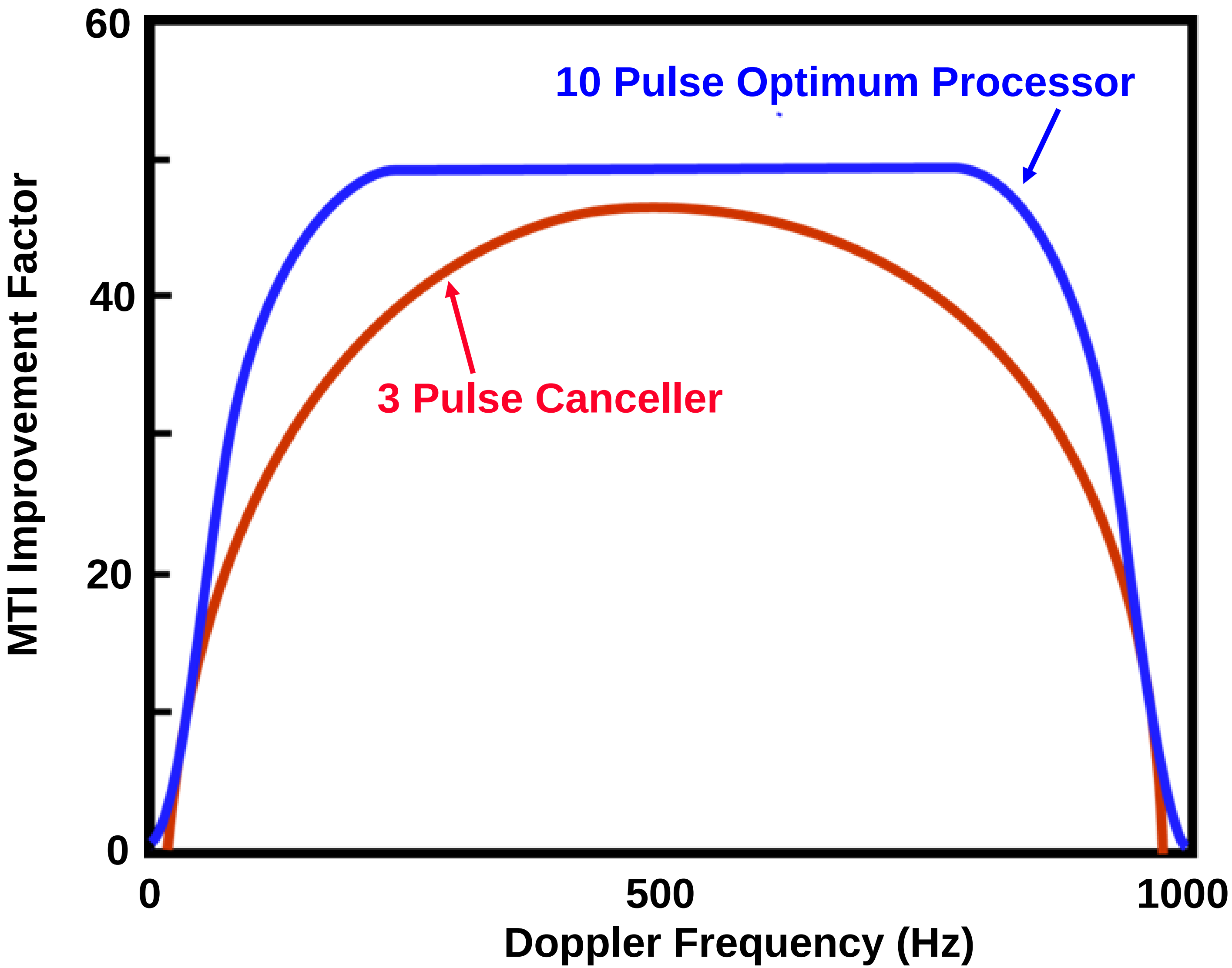
M = Number of pulses processed

See De Long et al, Reference 14 for detailed derivation

Pulse Doppler Processing

- The optimum weights are given by: $W_i^{OPT} = k \sum_{j=1}^{M} M_{ij}^{-1} S_j$
  - The optimum filter weights are a function of Doppler frequency
- In lecture 18, these issues will be studied in more detail.
  - Also, see Reference 9
    - It's a great, instructive readable reference for this material
- Because of the variable nature of ground clutter and rain, a simple high pass filter (MTI canceller) using a few (2-5) pulses will not come close to simultaneously rejecting both ground and rain clutter
  - At least 8 to 10 pulses are required for good rain rejection
  - Much of the rain clutter will pass through a high pass filter
- Typically, a set (bank) of Doppler filters are used, in parallel, to given good target detection over the range of Doppler frequencies
  - 0 to the PRF (Blind Speed)
  - The number of filters usually is equal to the number of pulses processed

MTI Improvement Factor Comparison

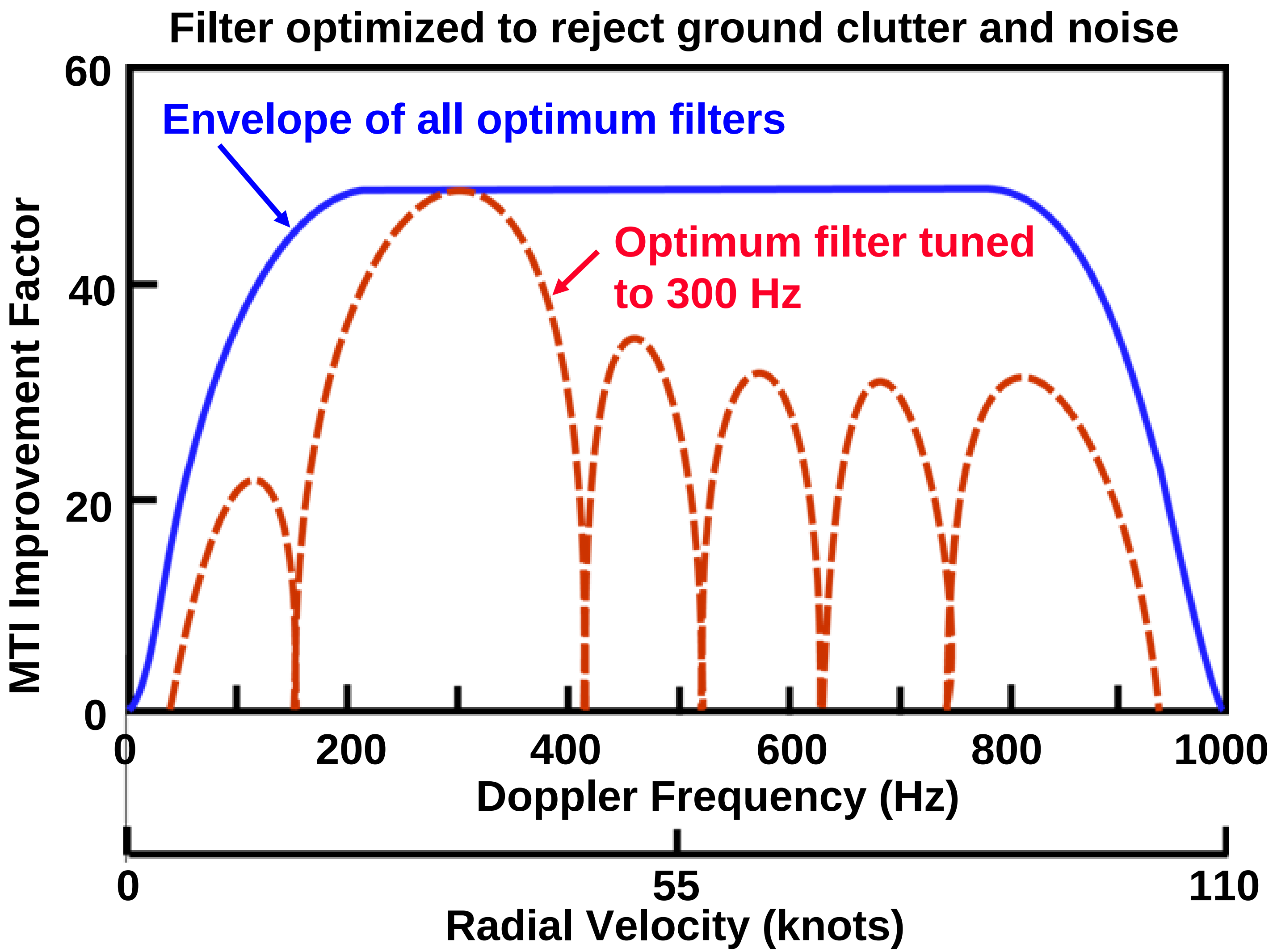


Typical Airport Surveillance Radar

Courtesy of Frank Sanders

Wavelength S-Band
10.7 cm
Antenna width 17.5 ft
Rotation rate 13 rpm
PRF 1000Hz
C/N =40 dB

MTI Improvement for One Optimum Filter



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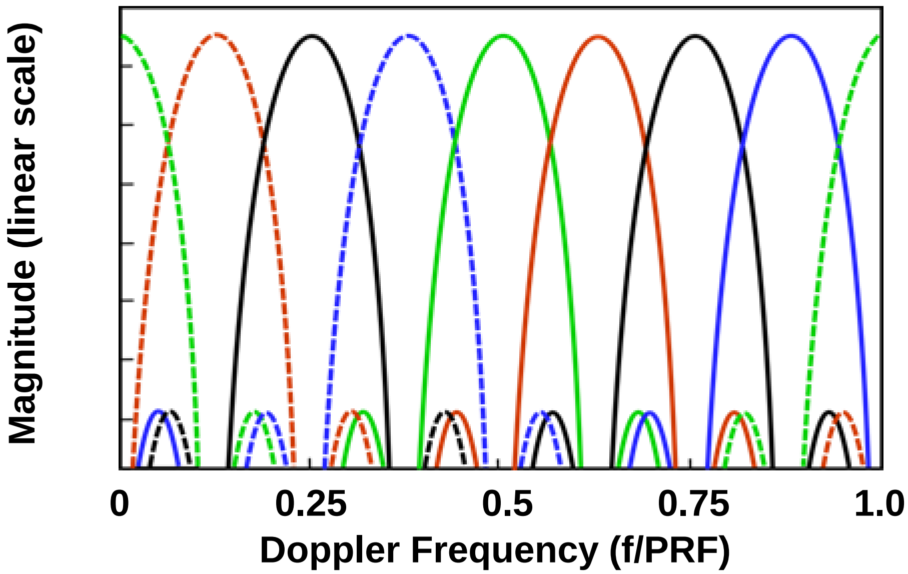
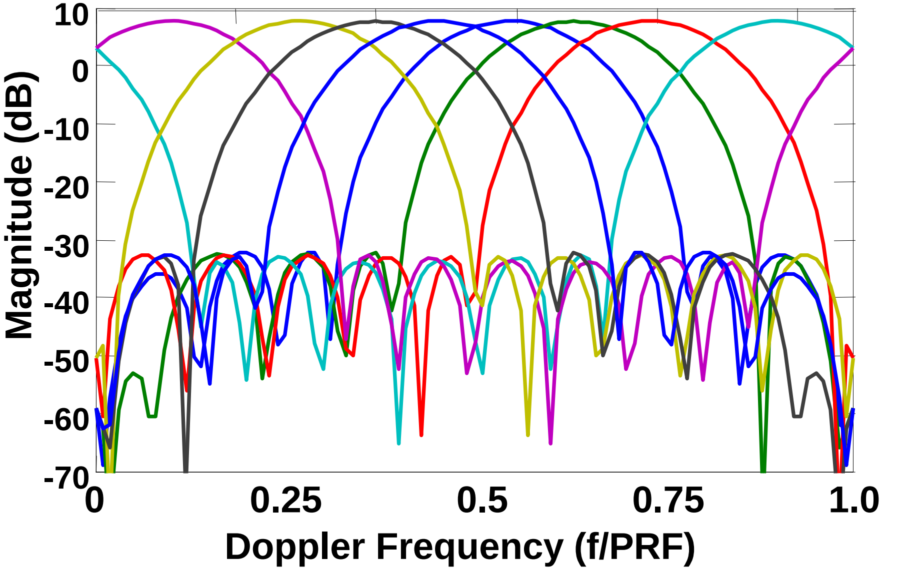
No. of pulses 10

Implementations of a Set of Doppler Filters



Doppler Filter Bank (linear scale)
8 Filters - DFT- 13 dB Sidelobes



Doppler Filter Bank
8 Filters - Shaped - Low Doppler Sidelobes

- The simplest way to implement a bank of filters is with a Discrete Fourier Transform (DFT)
  - Note the 13 dB sidelobes will give poor suppression of rain clutter
  - Weighting the input signal or use of other techniques, to be discussed in the next lecture, along with integrating an adequate number of pulses will give excellent target detection in the presence in even heavy rain

Viewgraph Courtesy of MIT Lincoln Laboratory
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Aircraft Surveillance Radar (ASR) Problem

Courtesy of MIT Lincoln Laboratory
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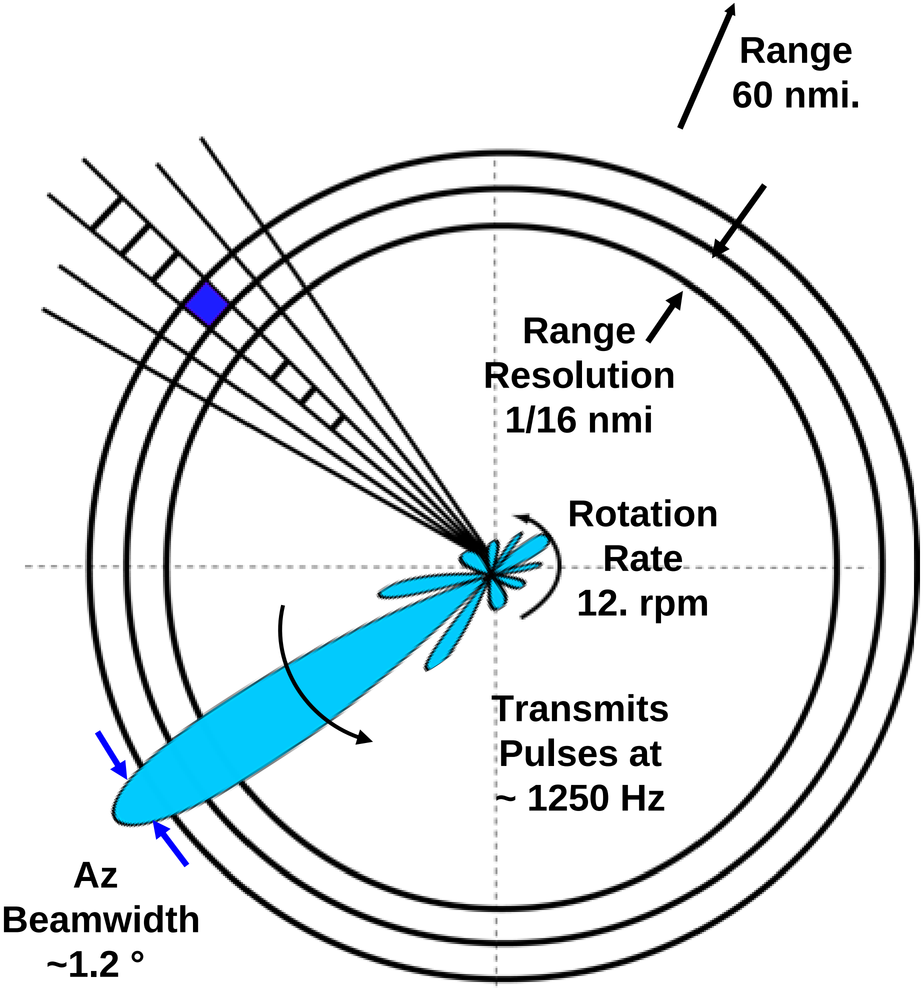


Range - Azimuth - Doppler Cells

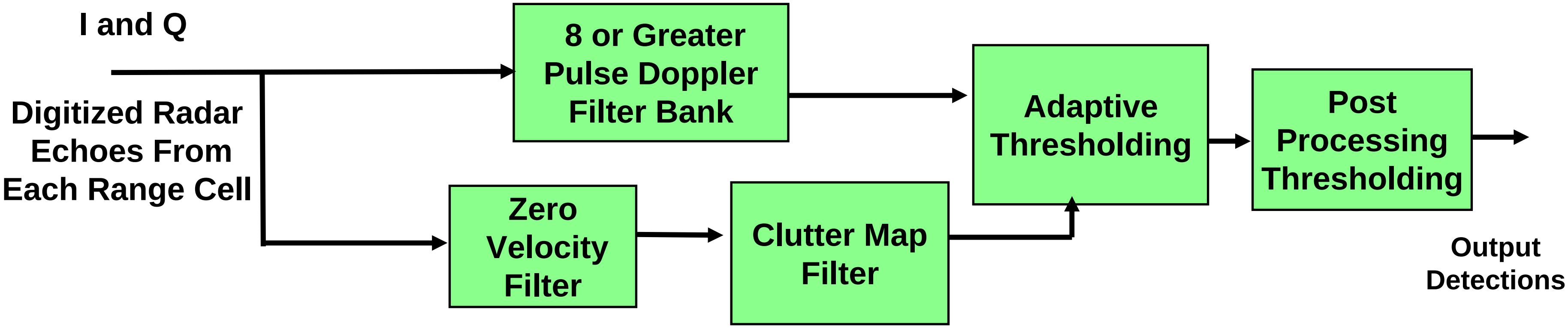
~1000 Range cells
~500 Azimuth cells
~8-10 Doppler cells
5,000,000 Range-Az-Doppler Cells
to be threshold every 4.7 sec.

As Antenna Rotates
~22 pulses / Beamwidth

Moving Target Detector (MTD) Processor

Issue	Solution
Ground Clutter	1. Eight Pulse Doppler Linear Coherent Filters (10 pulses)
Second Time Around Clutter	2. Coherent Transmitter 3. Constant PRF within coherent processing interval
Rain	4. Doppler Filter Bank 5. Adaptive Thresholding for each (Range Azimuth Doppler) Cell - 3.9 million cell
Tangential Targets, Blind Speeds	6. Fine Grained Clutter Map 7. Multiple PRFs

Moving Target Detector (MTD)



- Pulse Doppler filtering on groups of 8 or more pulses with a fine grained clutter map.
- Aircraft are detected in ground clutter and / or rain with the Doppler filter bank & use of 2 PRFs.
- Birds and ground traffic are rejected in post processing, using Doppler velocity and a 2nd fine grained clutter map

Viewgraph Courtesy of MIT Lincoln Laboratory
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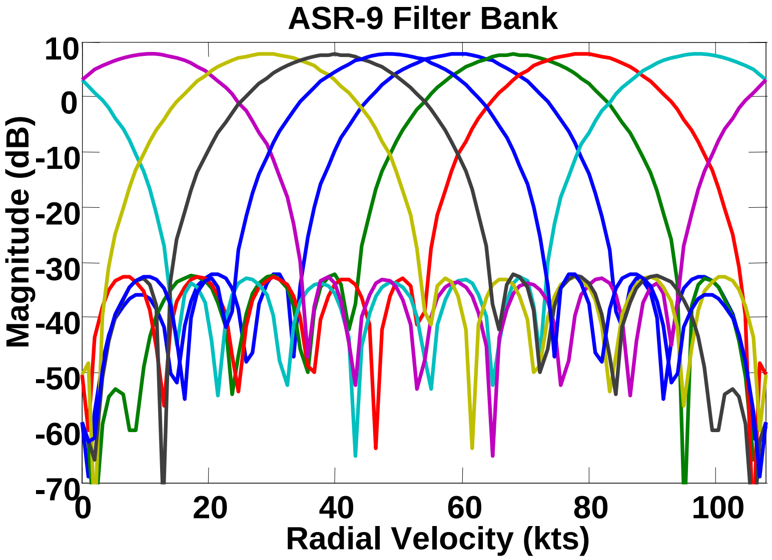
ASR-9 8-Pulse Filter Bank

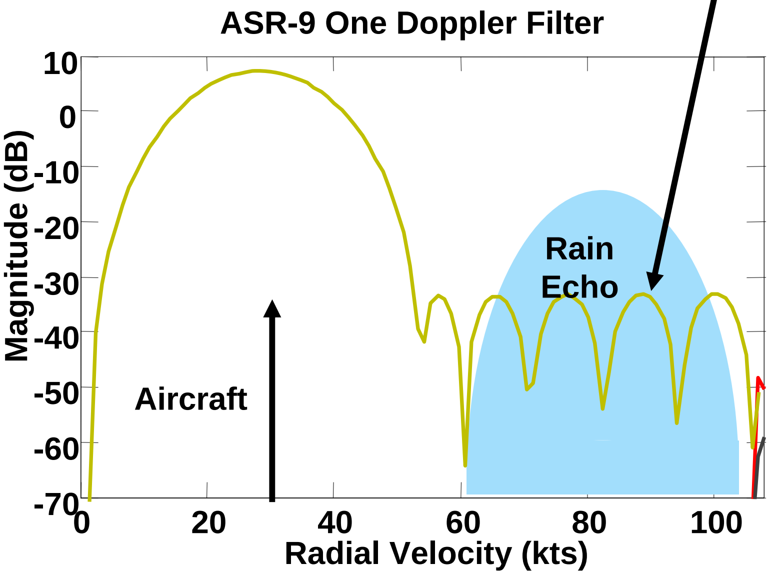
Courtesy of MIT Lincoln Laboratory

Note:

Doppler sidelobes 40 dB down from peak response



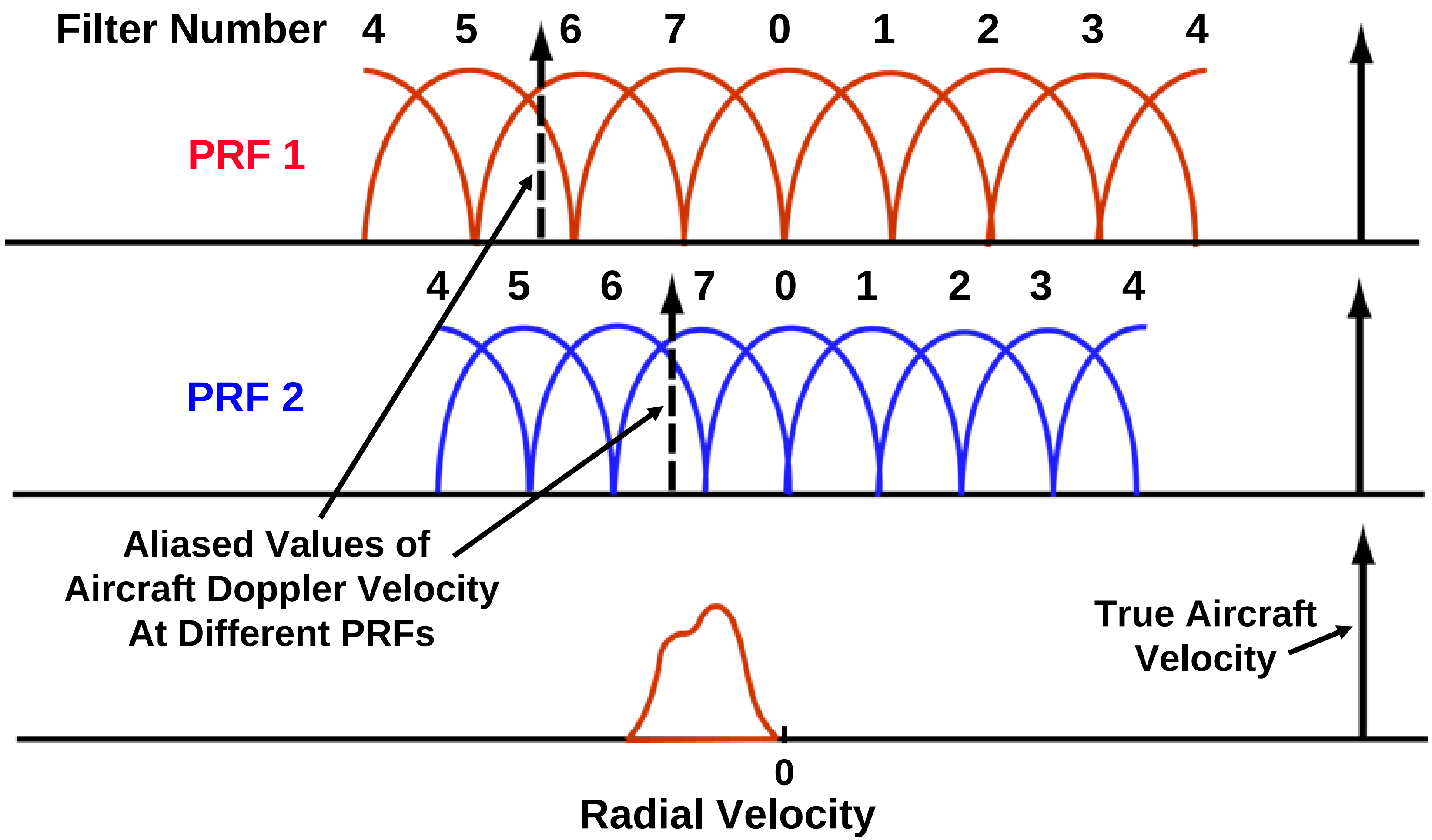
ASR-9 Filter Bank



ASR-9 One Doppler Filter

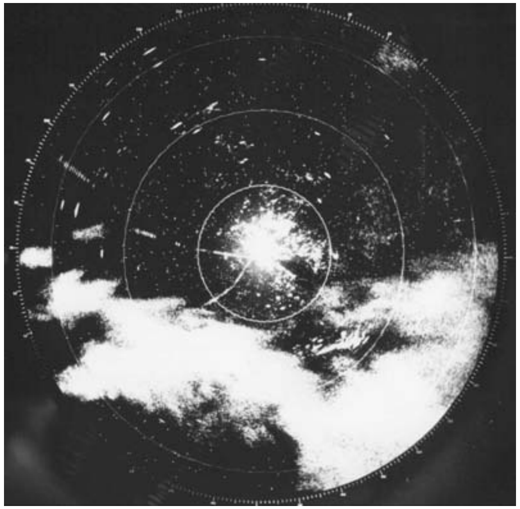
Viewgraph Courtesy of MIT Lincoln Laboratory Used with permission

Detection in Rain Using Two PRFs

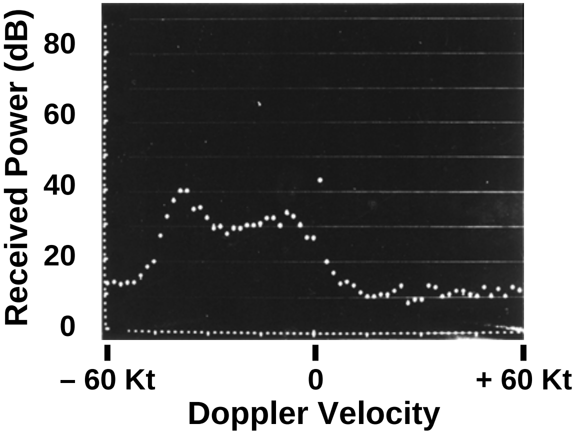


MTD Performance in Rain

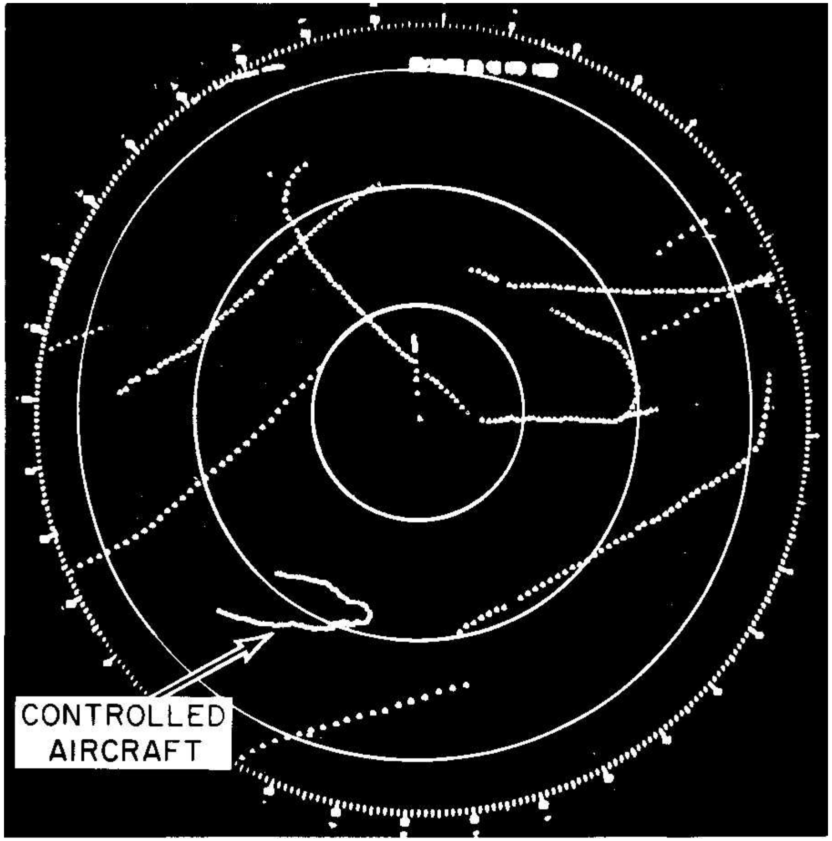
Unprocessed Radar Returns



Doppler Spectrum of Rain

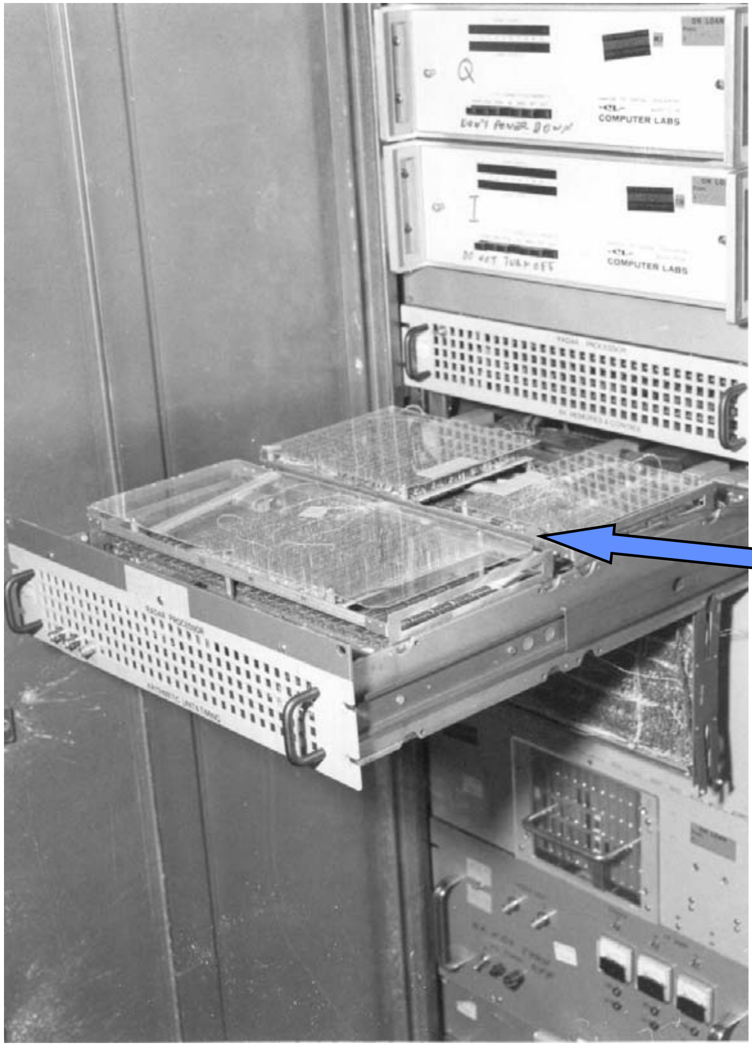


Time History of MTD Radar Tracker Output
August 1975, FAA Test Center



Photographs Courtesy
of FAA

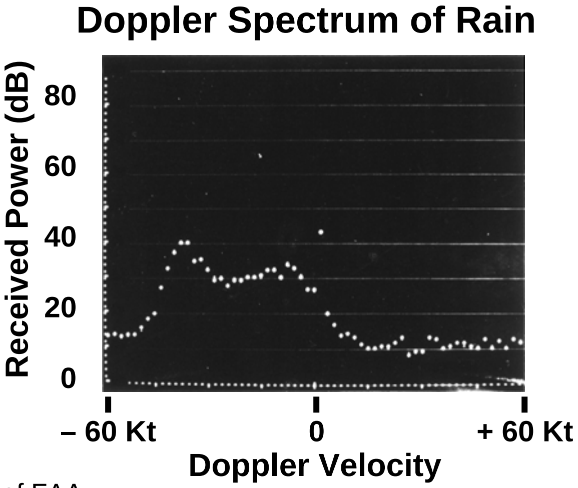
Moving Target Detector - I (1975)



Courtesy of FAA

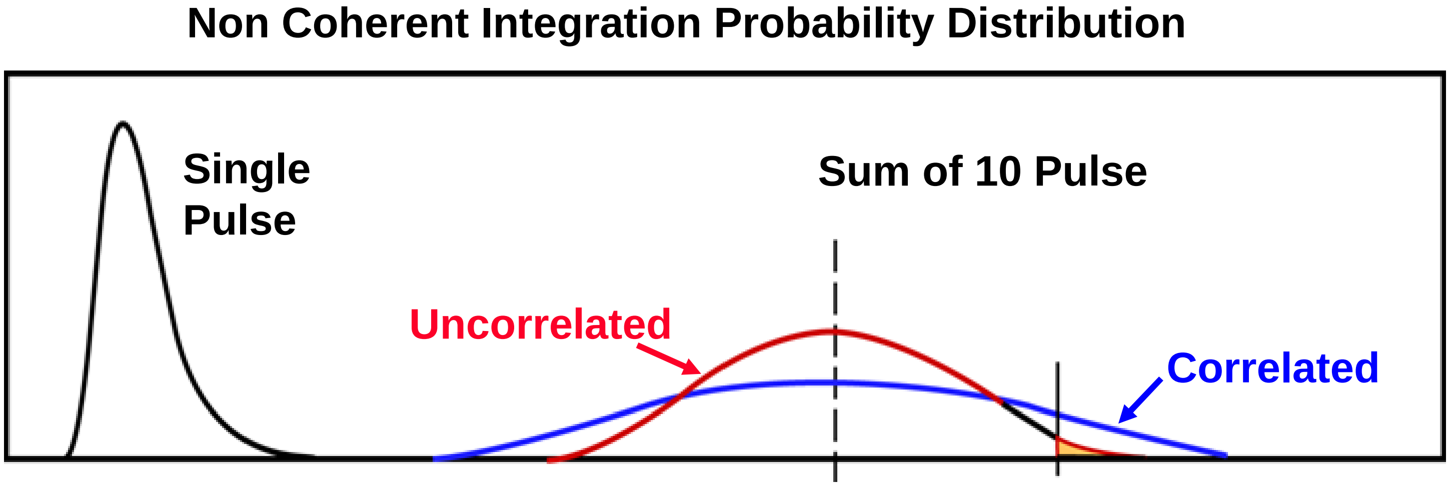
- A/D Converters
  - 10 Bit 2.6 MHz
  - Top one “Q”
  - Bottom one “I”
- Input Memory
  - Corner turning memory
- MTD Processor
  - ~1000 TTL Chips
- Clutter Map
  - Using “Drum” memory technology
- Analog IF Chassis

Non-Coherent Integration and the Effect of Correlated Clutter

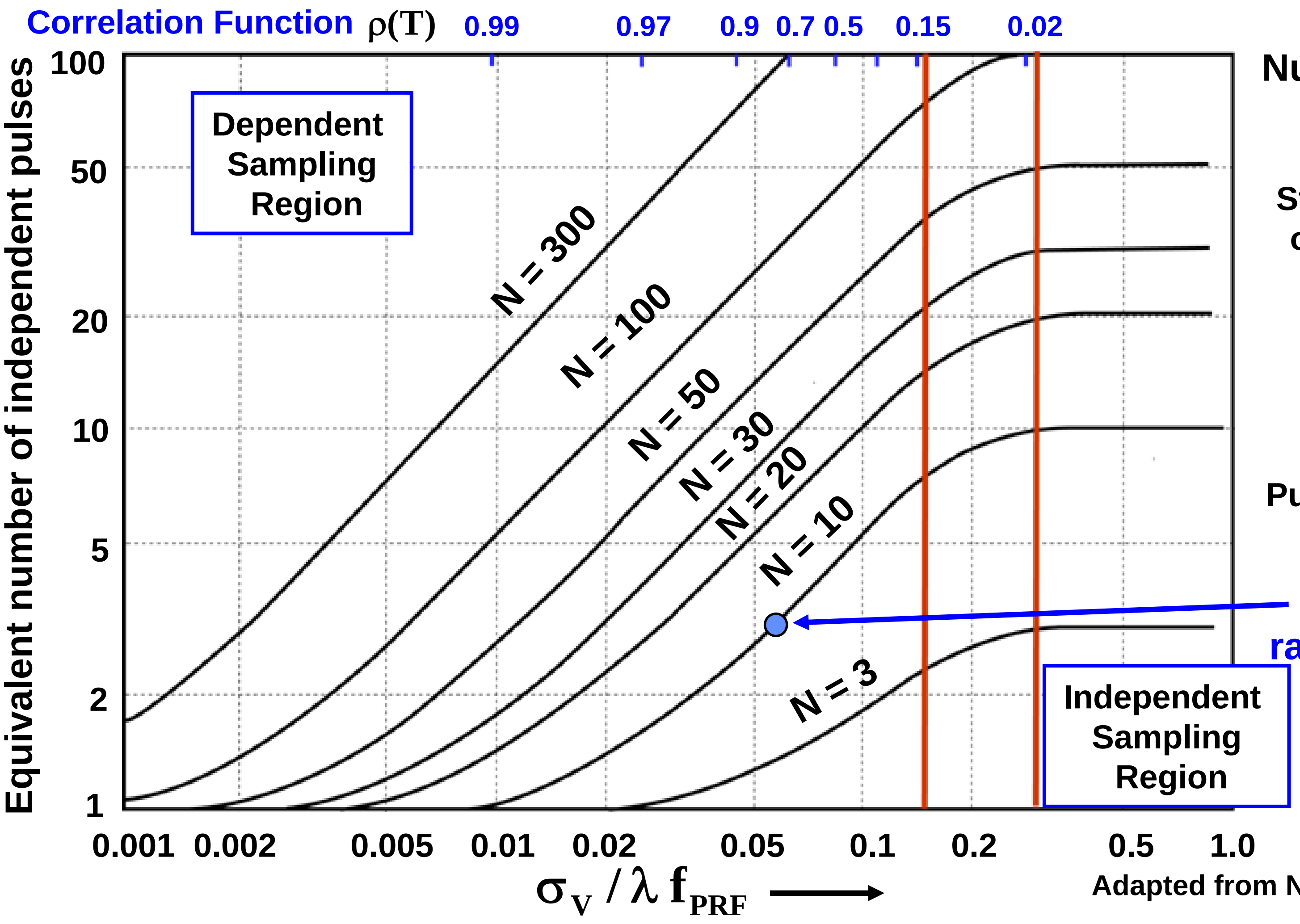


Courtesy of FAA

- Rain clutter residue that leaks through the MTI canceller is correlated from pulse to pulse
- Non coherent integration of correlated clutter residue is less efficient than with uncorrelated noise



Independent Pulses for Partially Correlated Waveforms



Adapted from Nathanson, Reference 5

N

Number of pulses

σ_V

Standard Deviation of Clutter Velocity

λ

Wavelength

$f_{PRF} = 1/T$

Pulse Repetition Rate

Blue dot is for an S band 2D radar in rain clutter

- **Non-coherent integration of partially correlated pulses can often be very inefficient**

MTD Implementations

- The first 2 versions of the MTD were designed, built for the FAA by MIT Lincoln Laboratory from the early to the late 1970s and are documented extensively*
- After operational testing of MTD II, These concepts were included into the specification of the ASR-9 and incorporated in that radar** along with additional improvements
- These concepts are presently implemented in almost all ground based low PRF radars and have influenced the extensive evolution of pulse Doppler processing onto sea based and airborne platforms, as further improvements in digital processing technology and algorithmic techniques have advanced

*See References 1, 8, 9 , 10, 11, 12 for extensive discussion

**See Taylor References 15

MTD Summary

- The MTD proved that, for the first time, digital signal processing hardware and algorithmic technology could be implemented in a manner that would give excellent aircraft detection while rejecting all forms of clutter (ground, rain, etc), under almost all conditions, so that radar and beacon reports could be reliably correlated and displayed to the air traffic controller.

- Solving this particular civilian problem has been, over the ensuing years, a catalyst, for the appropriate application of this general approach to many other civilian and military radar problems:
  - Understanding that Moore's law will allow cost effective use, in the near future, of processing techniques, seemingly not cost effective today
    - Some experts said " You can never make wire wrapped 1000 IC work reliably (Incidentally, they were wrong!)
    - Now that processing can be done with a few programmable Power PC cards
  - Integration of many pulses to use low Doppler sidelobes to reject moving clutter (rain, chaff , sea clutter, etc.)
  - Use of high resolution clutter maps, to detect tangential targets
  - Solving the "signal processing to radar target display" problem in an integrated manner

MTD Clutter Map Techniques

- Clutter maps are a memory which stores for each range-CPI cell in the radar’s coverage the value of the noise and clutter echo in that cell
  - Clutter maps are usually implemented using a recursive filter
  - For each range – CPI cell, the clutter map is updated using the following algorithm

$$A(n+1)=\frac{1}{N}\left(A(n)\right)+\left(1-\frac{1}{N}\right)\left(A(n-1)\right)$$

  - N = 8 for the MTD   n = scan number

- They are used to detect targets whose radial velocity is at of near zero and whose backscatter echo is greater than the clutter and / or noise amplitude stored in the clutter map
  - The clutter map channel offers a method of detecting targets that are not detected by the subset of the Doppler filters, that are adjacent to zero Doppler and whose shape is designed to strongly reject ground echoes near zero Doppler

Clutter Map Thresholding

- **Clutter map detection techniques use temporal thresholding techniques**
  - **Spatial CFAR techniques would detect the edges of moving rain clouds**
- **Target detection is declared if the size of the average of the coherently integrated return is M times the previous scan's value, which is stored on the clutter map**
- **This process is performed for each Range CPI cell every scan of the radar**
  - **~350,000 cell for an ASR radar**
- **Additional Points**
  - **This technique makes possible detection of tangential aircraft flying tangentially near large discrete pieces of ground clutter**
    - **Called "Inter-clutter visibility" in the literature**
  - **Aircraft moving tangentially to the radar are give large specular echoes, which enhances this detection mode**

Post Signal Processing Clutter Map Techniques

- Even with these, relatively sophisticated signal processing and thresholding techniques, performed on single range – CPI basis, sometimes excessive false detections do occur

- These can be caused by
  - Heavy bird migration
  - Ground clutter whose echoes exceed the A/D dynamic range
  - Automobile traffic
  - And other sources

- More sophisticated Area CFAR very similar to clutter maps have been developed to effectively deal with these problems
  - This set of thresholding techniques are employed before the tracking function
  - Good places to learn more detail about these "post processing" techniques are detailed ;
    - References 11 and 12;
    - Reference 6, pp 284-285

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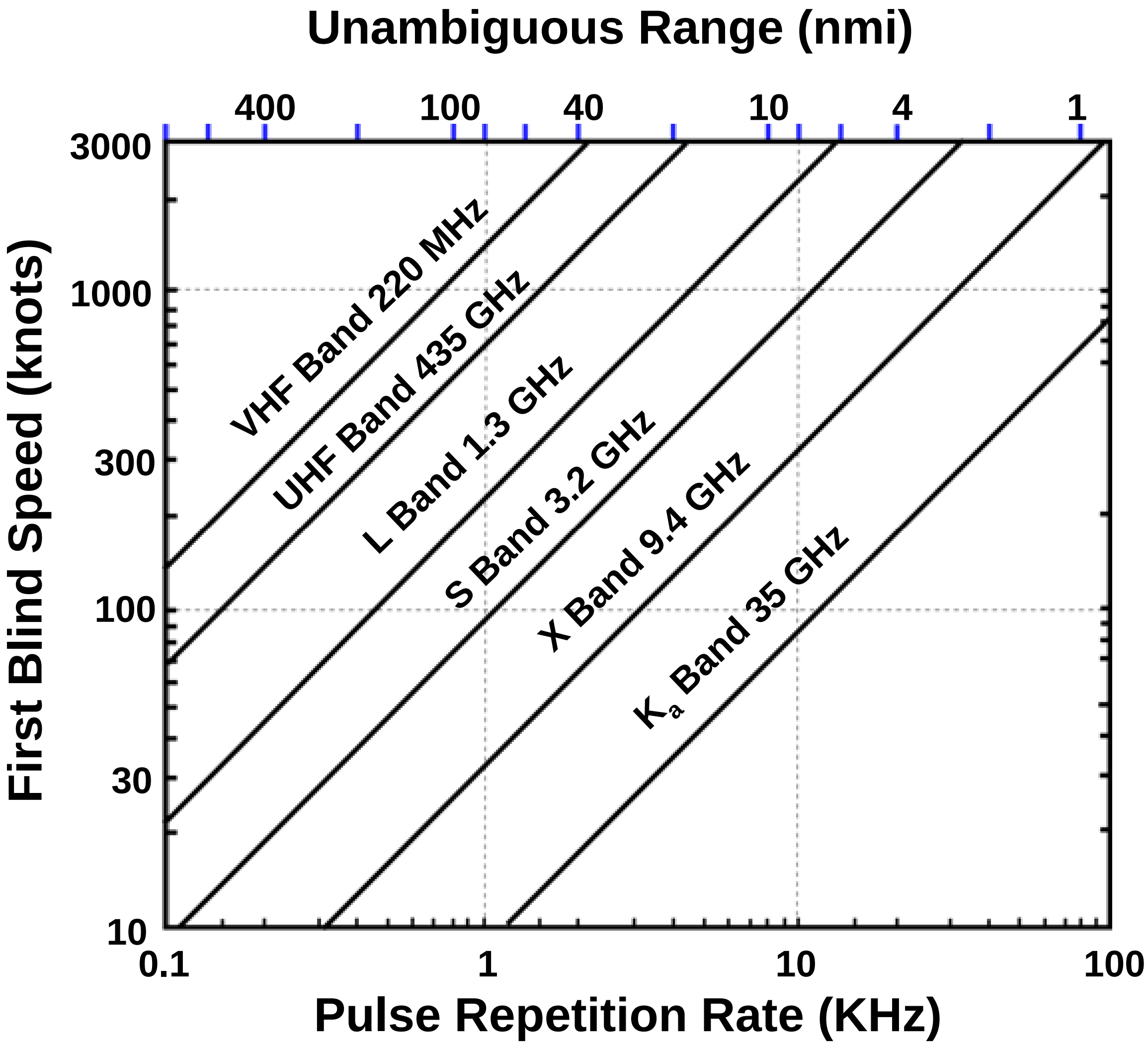
Unambiguous Doppler Velocity and Range



$$V_B = \frac{\lambda f_{PRF}}{2}$$

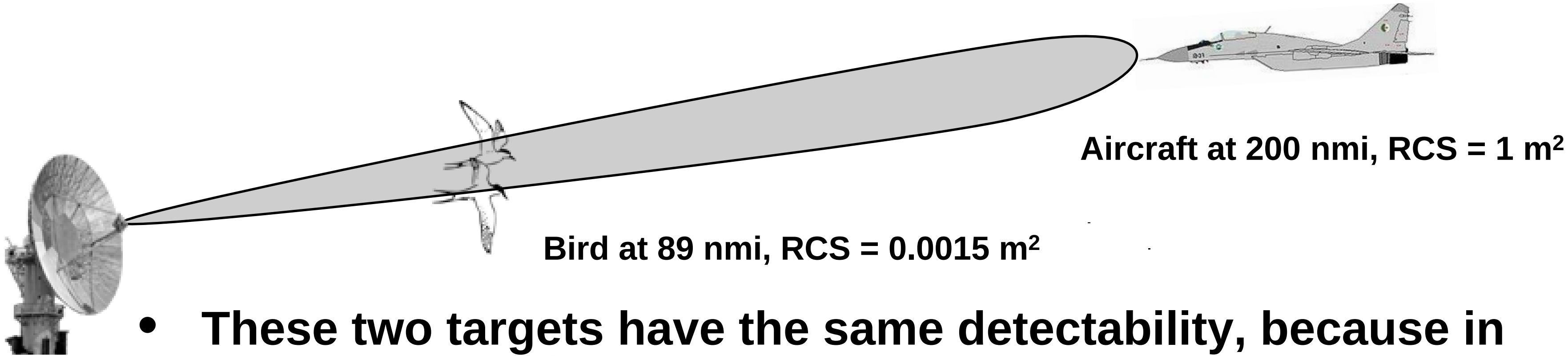
and

$$R_U = \frac{c}{2 f_{PRF}}$$

Yields

$$\boxed{V_B = \frac{\lambda c}{4 R_U}}$$

Sensitivity Time Control



- These two targets have the same detectability, because in the radar equation: $\frac{S}{N} \propto \frac{\sigma}{R^4}$
- This false target issue can be mitigated by attenuating to the received signal by a factor which varies as 1/R^4
  - Can also be accomplished by injecting noise into the receive channel , which falls off as 1/R^4
- Radars that utilize range ambiguous waveforms, cannot use STC, because long range targets which alias down in range, would be adversely attenuated by the STC
  - For these waveforms, other techniques must be used to mitigate the false target problem due to birds

Classes of MTI and Pulse Doppler Radars

	Low PRF	Medium PRF	High PRF
Range Measurement	Unambiguous	Ambiguous	Very Ambiguous
Velocity Measurement	Very Ambiguous	Ambiguous	Unambiguous

Low PRF

- Wind blown clutter may be a problem
- Can use STC

Medium PRF

- Wind blown clutter may be a problem
- Range eclipsing losses
- Far out targets compete with near in clutter
- Can't use STC
- Ambiguities difficult to remove

High PRF

- Range eclipsing losses
- Distant targets compete with near in clutter
- Can't use STC

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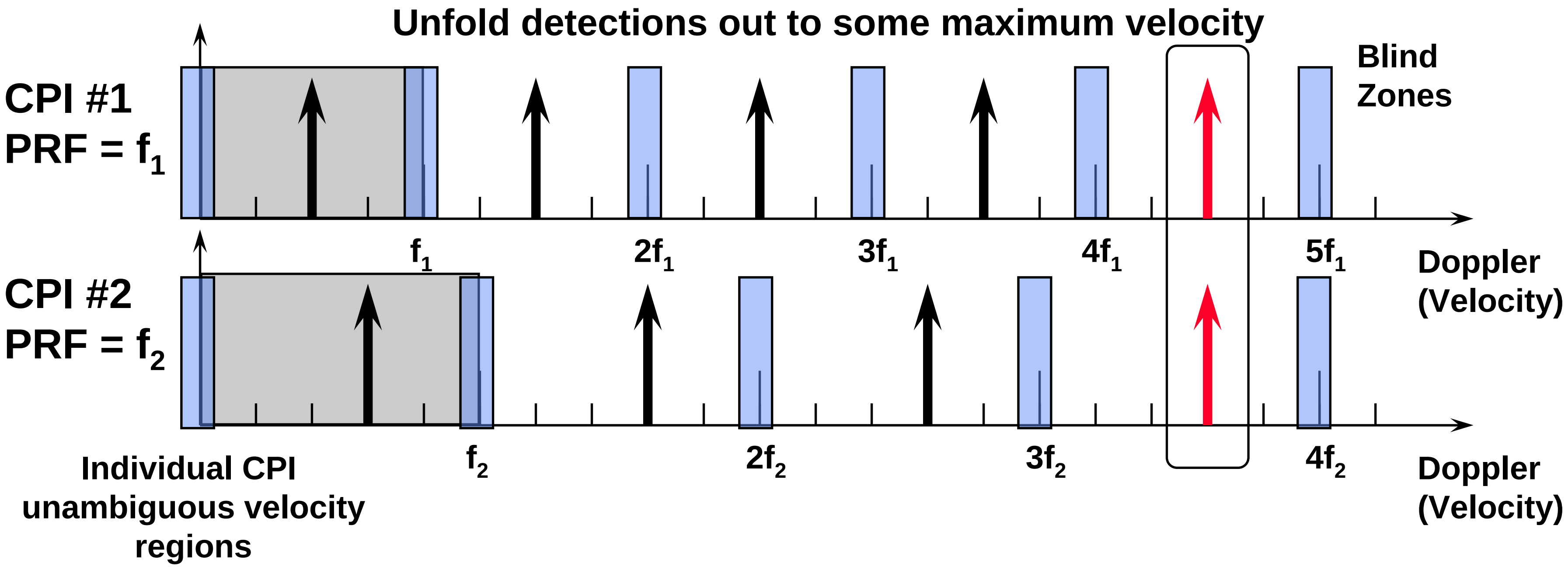
Velocity Ambiguity Resolution



- Split dwell into multiple CPIs at different PRFs
  - Scan to scan, even pulse-to-pulse changes also possible
- Moves blind velocities to ensure detection of all non-zero velocity targets
- True target velocity is where best correlation across CPIs occurs
- Choose PRFs so that least common multiple occurs above desired maximum unambiguous velocity

Viewgraph Courtesy of MIT Lincoln Laboratory
Used with permission

Chinese Remainder Theorem

R_c = ($C_1A_1 + C_2A_2 + C_3A_3$) modulo ($m_1\ m_2\ m_3$)

(assumes 3 PRFs)

R_c = True range/Doppler cell number

Cell number is range expressed in pulse widths or Doppler velocity expressed in Doppler filter widths

A_i = Ambiguous range or Doppler cell number for i^{th} PRF

$PRF_i = 1 / t\ m_i$ t = pulsewidth

$m_1\ m_2\ m_3$ are relatively prime numbers

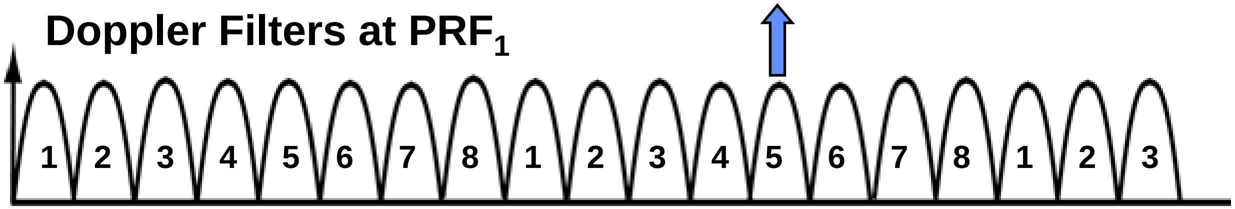
C_1 C_2 and C_3 are related to $m_1\ m_2$ and m_3 by

$C_1 = b_1 \times m_2\ m_3 = 1$ modulo m_1
$C_2 = b_2 \times m_3\ m_1 = 1$ modulo m_2
$C_3 = b_3 \times m_1\ m_2 = 1$ modulo m_3

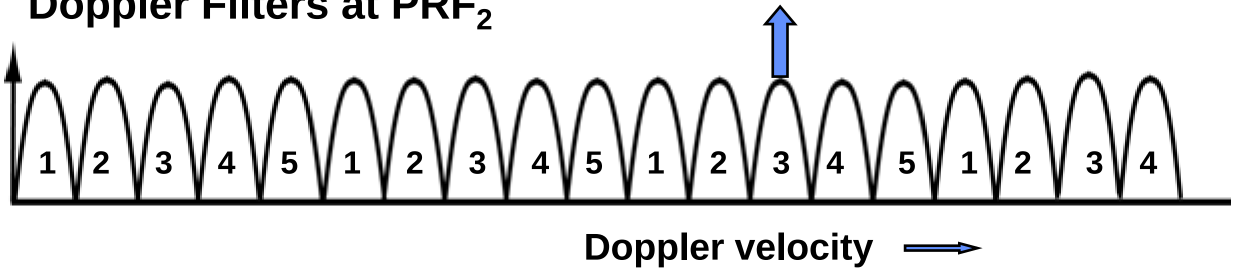
where b_1 = smallest positive integer which, when multiplied by $m_2\ m_3$ and divided by m_1 gives unity as the remainder

Example - Chinese Remainder Theorem

Doppler Filters at PRF_1



Doppler Filters at PRF_2



S Band Radar

PRF_1 = 800Hz
Blind Sped = 80 knots

PRF_1 = 500Hz
Blind Speed = 50 knots

Each filter 100 Hz wide
(10 knots)

$A_1 = 5$ $m_1 = 8$
$A_2 = 3$ $m_2 = 5$

$$
\begin{aligned}
R_c &= (C_1A_1 + C_2A_2) \text{ modulo } (40) \\
&= \left[5(5x5) + 3(8x2)\right] \text{ modulo } (40) \\
&= (125+48) \text{ modulo } (40) \\
&= 173 \text{ modulo } (40) \\
&= 13
\end{aligned}
$$

True velocity = 130 knots

$C_1 = b_1\, m_2 = 1$ modulo m_1
$C_2 = b_2\, m_1 = 1$ modulo m_2

$C_1 = b_1\, m_2 = 1$ modulo m_1
$C_1 = b_1\, 5 = 1$ modulo 8
$b_1 = 5$

$C_2 = b_2\, 8 = 1$ modulo 5
$b_2 = 2$

IEEE New Hampshire Section
IEEE AES Society

Example - Chinese Remainder Theorem

Shoe Length of 4 Men's Feet		Measure of a Room (Remainder)	
Bob	$m_1 = 7$ inches	Bob	2 inches remainder = A_1
Larry	$m_2 = 8$ inches	Larry	5 inches remainder = A_2
Moe	$m_3 = 9$ inches	Moe	5 inches remainder = A_3
Curly	$m_4 = 11$ inches	Curly	6 inches remainder = A_4

WHAT IS THE LENGTH OF THE ROOM ??

$L = (C_1A_1 + C_2A_2 + C_3A_3 + C_4A_4) \text{ modulo } (m_1 m_2 m_3 m_4)$

$m_1 m_2 m_3 m_4 = 5544$

$C_1 = b_1 \times m_2 m_3 m_4 = 1 \text{ modulo } m_1$

$b_1 \times 8 \times 9 \times 11 = 1 \text{ modulo } 7$

$b_1 \times (7+1) \times (7+2) \times (7+4) = 1 \text{ modulo } 7$

$8 b_1 = 1 \text{ modulo } 7$

$b_1 = 1$

$$L = \left[A_1(792\text{x}1) + A_2(693\text{x}5) + A_3(616\text{x}7) + A_4(504\text{x}5)\right] \text{ modulo } 5544$$

$$= \left[2(792) + 5(3465) + 5(4312) + 6(2520)\right] \text{ modulo } 5544$$

$$= \left[1584 + 17,325 + 21,560 + 15,120\right] \text{ modulo } 5544$$

$$= 149 \text{ inches}$$

Outline

- Introduction

- Problem perspective
  - Burst Waveforms and their properties
  - The impact of Moore’s Law on radar Signal Processing

    Past, present, and the future

- Pulse Doppler Processing Techniques
  - Description of pulse Doppler processing
  - Low PRF Example – Moving Target Detector (MTD)
  - Range and Doppler Ambiguities
  - Ambiguity Resolution - Chinese remainder theorem
  - The “Ambiguity Function”
  - Preview of Airborne Pulse Doppler issues

- Summary

Quick Matched Filter Review

- Matched Filter is the cross correlation between :
  - Received signal (plus noise) , and
  - A replica of the transmitted signal

$$s(t) = u(t)\, e^{2\pi j f_T t}$$

$$\text{Matched Filter Output} = \int_{-\infty}^{\infty} s_R(t)\, s^*(t - T_R)\, dt$$

T_R = Round trip time delay to target

- For low S/N assumed:
  - Autocorrelation of transmitted signal
  - It is assumed that Doppler velocity of target is zero
- Usually the target is moving and the Doppler frequency of the target is not zero
- Then, the output of matched filter is the cross correlation of the transmitted signal and the received Doppler shifted echo.

The Ambiguity Function

- The Ambiguity Function is the squared magnitude of the cross correlation (output of matched filter) of the transmitted signal and the received Doppler shifted echo.

- Studying (analytically and graphically) the two dimensional properties of the Ambiguity Function as both :
  - Time delay (range), and
  - Doppler frequency (Doppler velocity)

  are varied, can give great insight into understanding many of the waveforms properties, in particular:
  - Target resolution,
  - Waveform measurement accuracy,
  - Response to various types of clutter, and
  - Ambiguities in Doppler velocity and range

The Ambiguity Function

- The **Ambiguity Function** is the squared magnitude of the **cross correlation (output of matched filter) of the transmitted signal and the received Doppler shifted echo**.

- Thus, with some algebraic manipulation *

$$\chi(T_R, f_D) = \int_{-\infty}^{\infty} u(t)\, u^*(t + T_R)\, e^{2\pi j f_D t} dt$$

- Thus, the ambiguity function is $\left|\chi(T_R, f_D)\right|^2$
  - T_R is the round trip time delay to the target
  - f_D is the Doppler shift of the target
  - and $s(t) = u(t)\, e^{2\pi j f_T t}$

---

* See Skolnik Reference 1, pp 329-330 for details

Properties of the Ambiguity Function

- **Maximum value of the ambiguity function** $= (2E)^2$
  - **At true location of target** $T_D = 0$
  - **When,** $f_D = 0$

Note: $s(t) = u(t)e^{2\pi j f_0 t}$

- **Total volume under surface of ambiguity function** $= (2E)^2$

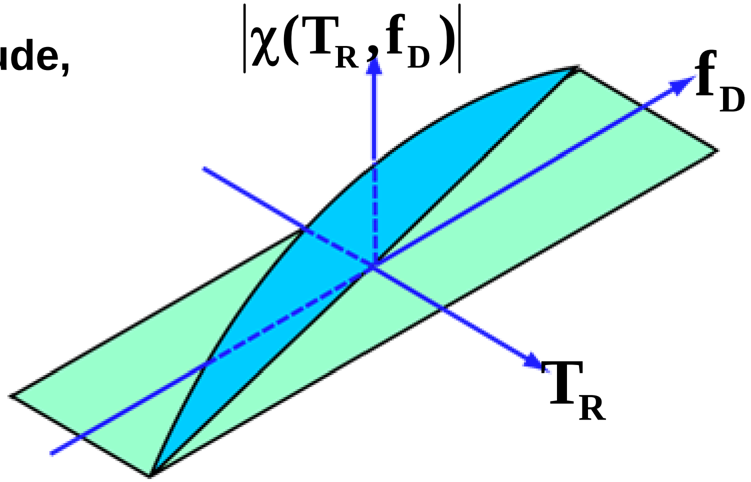
- **Behavior along** T_R **axis** $|\chi(T_R, 0)|^2 = \left|\int u(t)u^*(t+T_R)\,dt\right|^2$
  - **Square of autocorrelation function of** $u(t)$

- **Behavior along frequency,** f_D **, axis** $|\chi(0, f_D)|^2 = \left|\int u^2(t)e^{2\pi j f_0 t}\,dt\right|^2$
  - **Square of inverse Fourier Transform of** $u^2(t)$

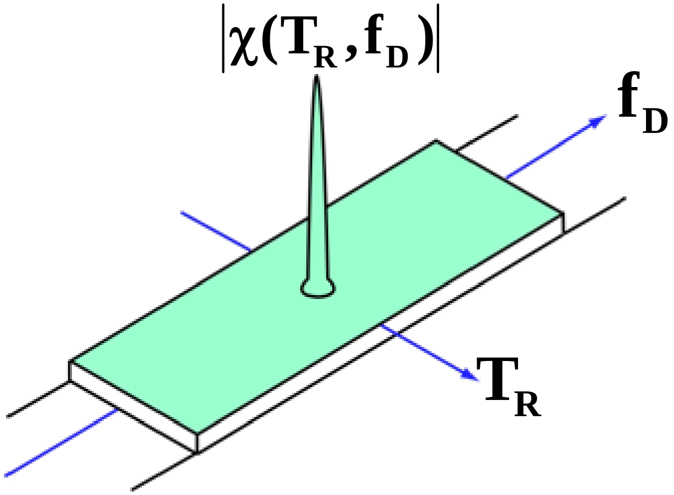
- **A good model of the ambiguity function, suggested by Skolnik, is a "box of sand"**
  - **Total volume of sand is** $= (2E)^2$**, The sand may be in different piles, but its volume is constrained to be** $= (2E)^2$

Three General Classes of Ambiguity Functions

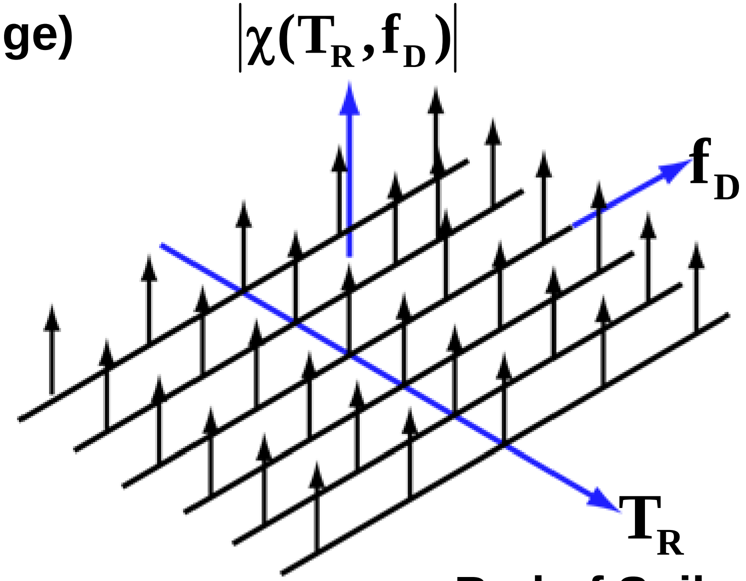
Typically, the magnitude, not the magnitude squared is plotted



Knife Edge (ridge)



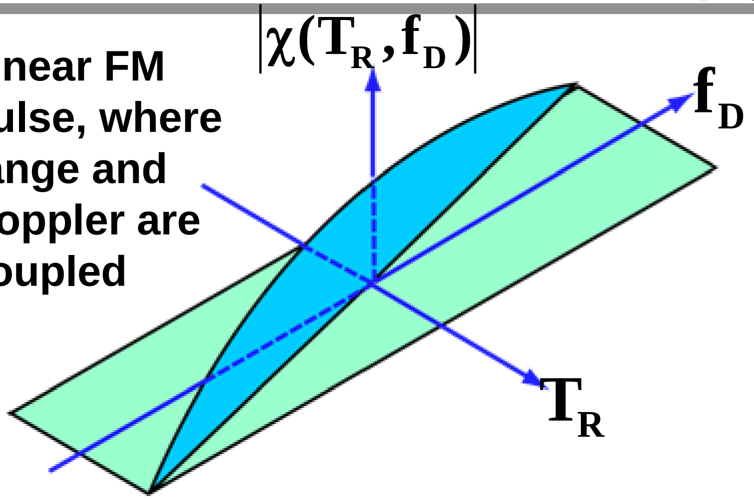
Thumbtack



Bed of Spikes

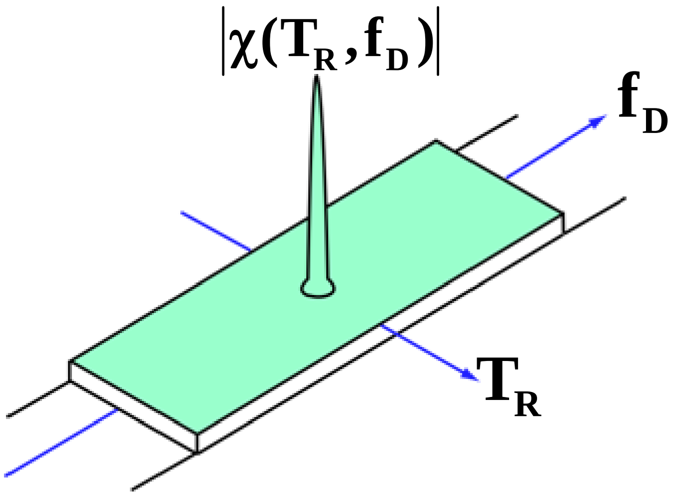
Three General Classes of Ambiguity Functions

Linear FM pulse, where range and Doppler are coupled



Knife Edge (ridge)

- **Knife Edge (ridge)**
- **Used to measure one parameter: range , Doppler, or a linear combination of range and Doppler**
- **Examples : a single rectangular pulsed sine wave or a single rectangular linear FM pulse**



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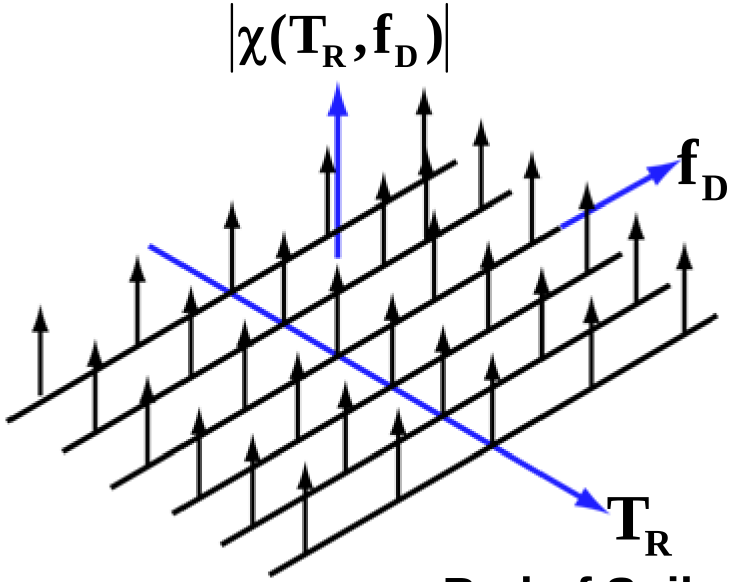
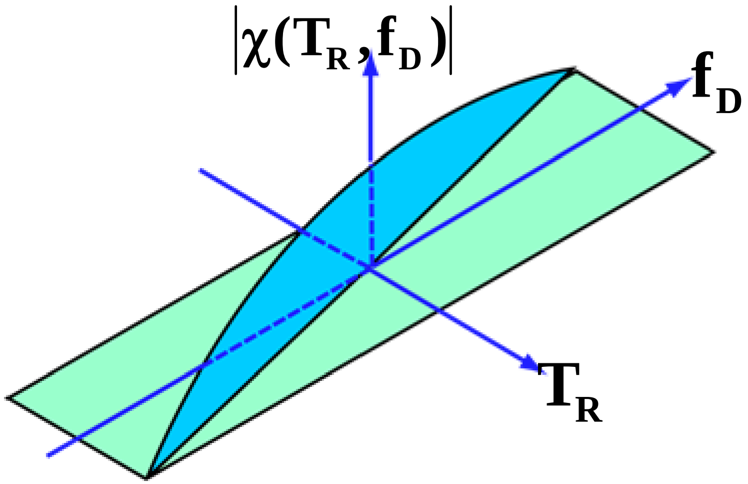


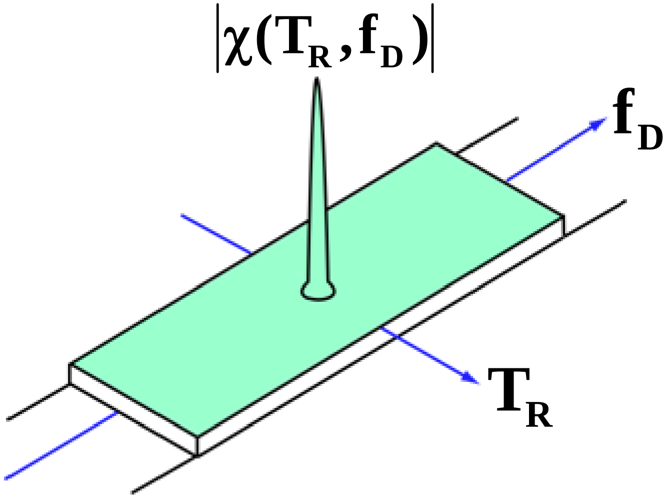
Bed of Spikes

Three General Classes of Ambiguity Functions



Knife Edge (ridge)

- Bed of Spikes
- Used to measure both range , Doppler with ambiguities
- Example : a burst of N pulses of sine wave



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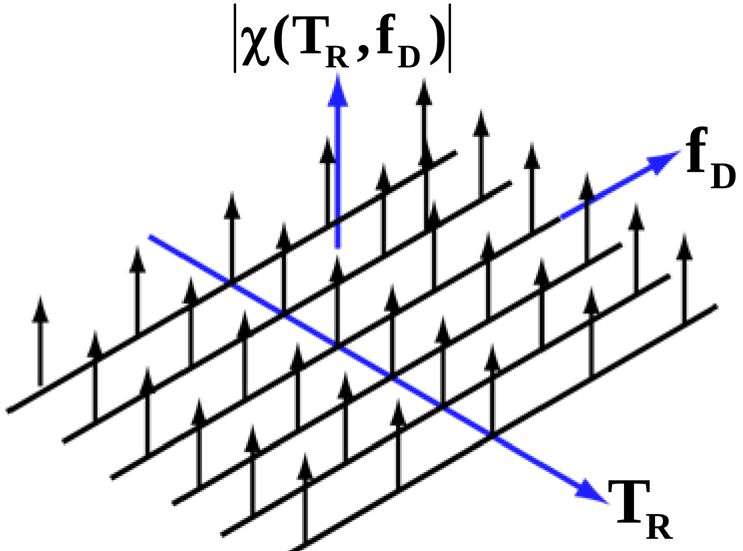
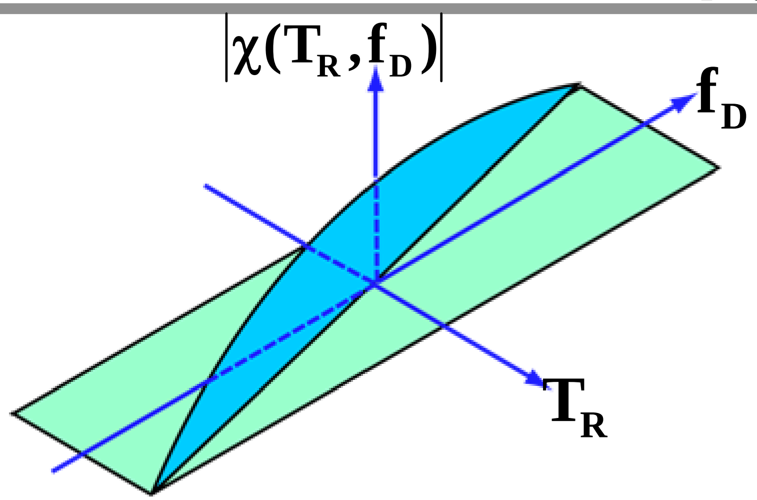


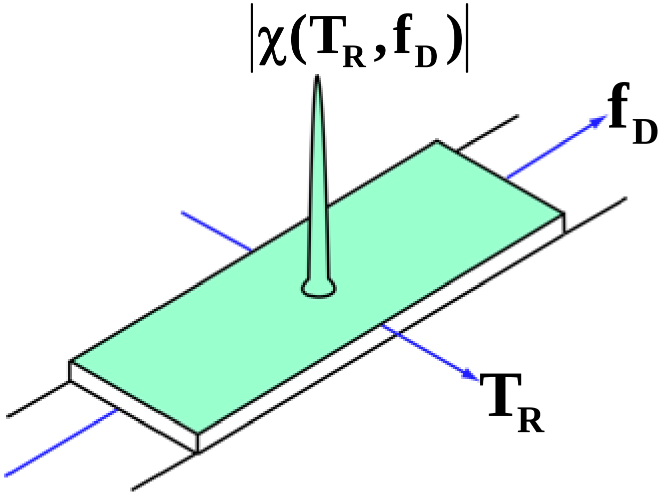
Bed of Spikes

Three General Classes of Ambiguity Functions

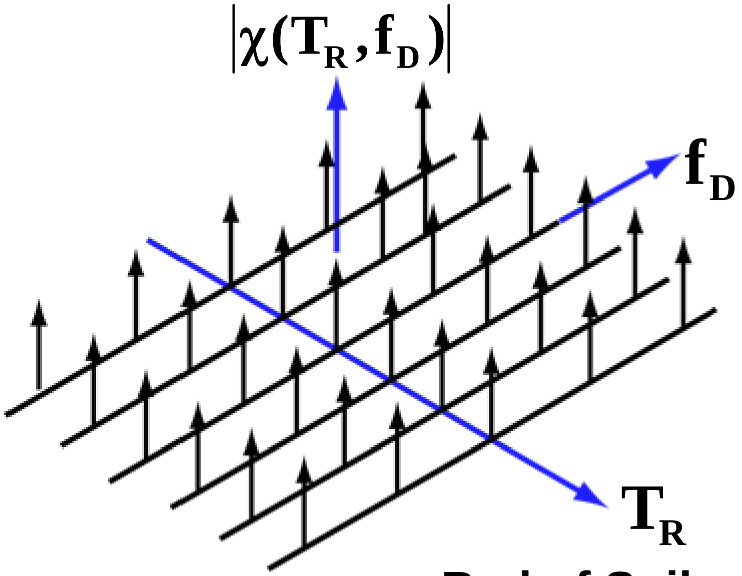
- **Thumbtack**
- **Examples : pseudorandom noise waveforms (rarely used in radar)**



Knife Edge (ridge)



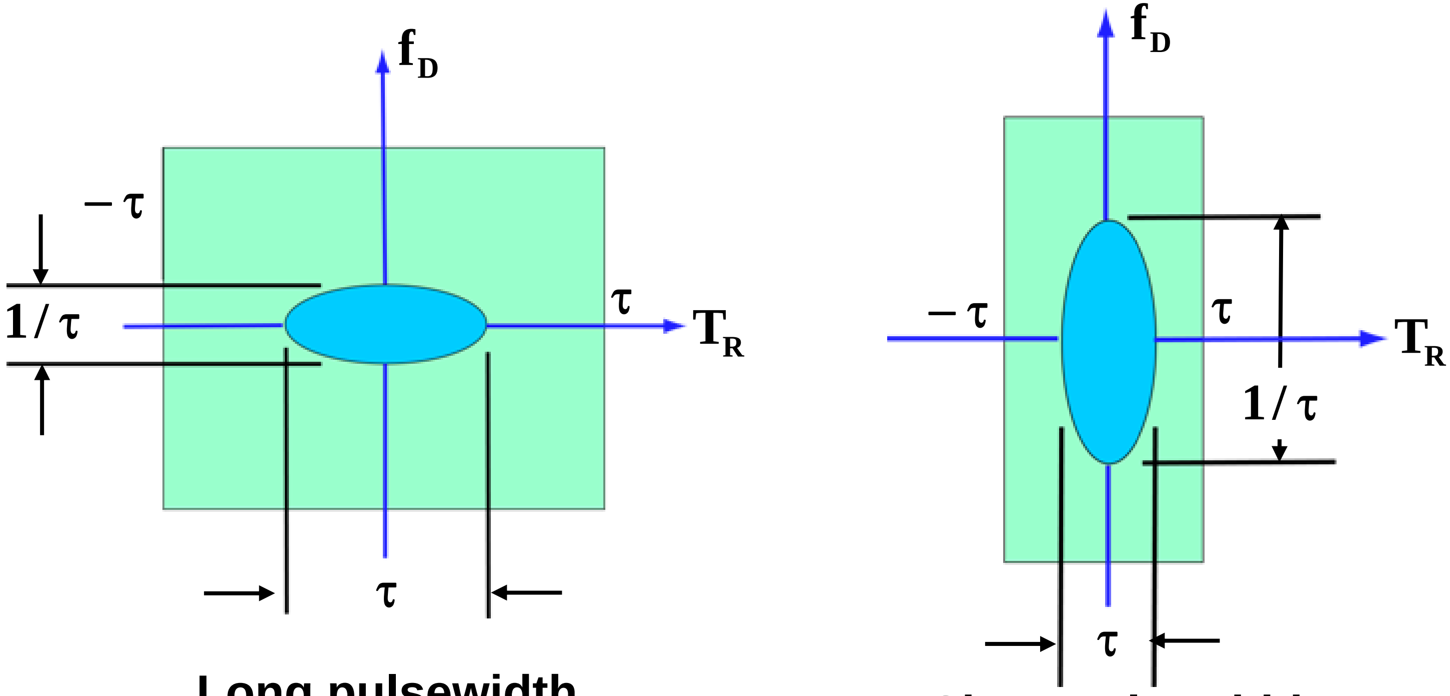
Thumbtack



Bed of Spikes

Ambiguity Function – Rectangular Pulse

- Ambiguity Function for two simple single sine wave pulses, each with different pulse widths
- Examples - 2D slices across Ambiguity Function



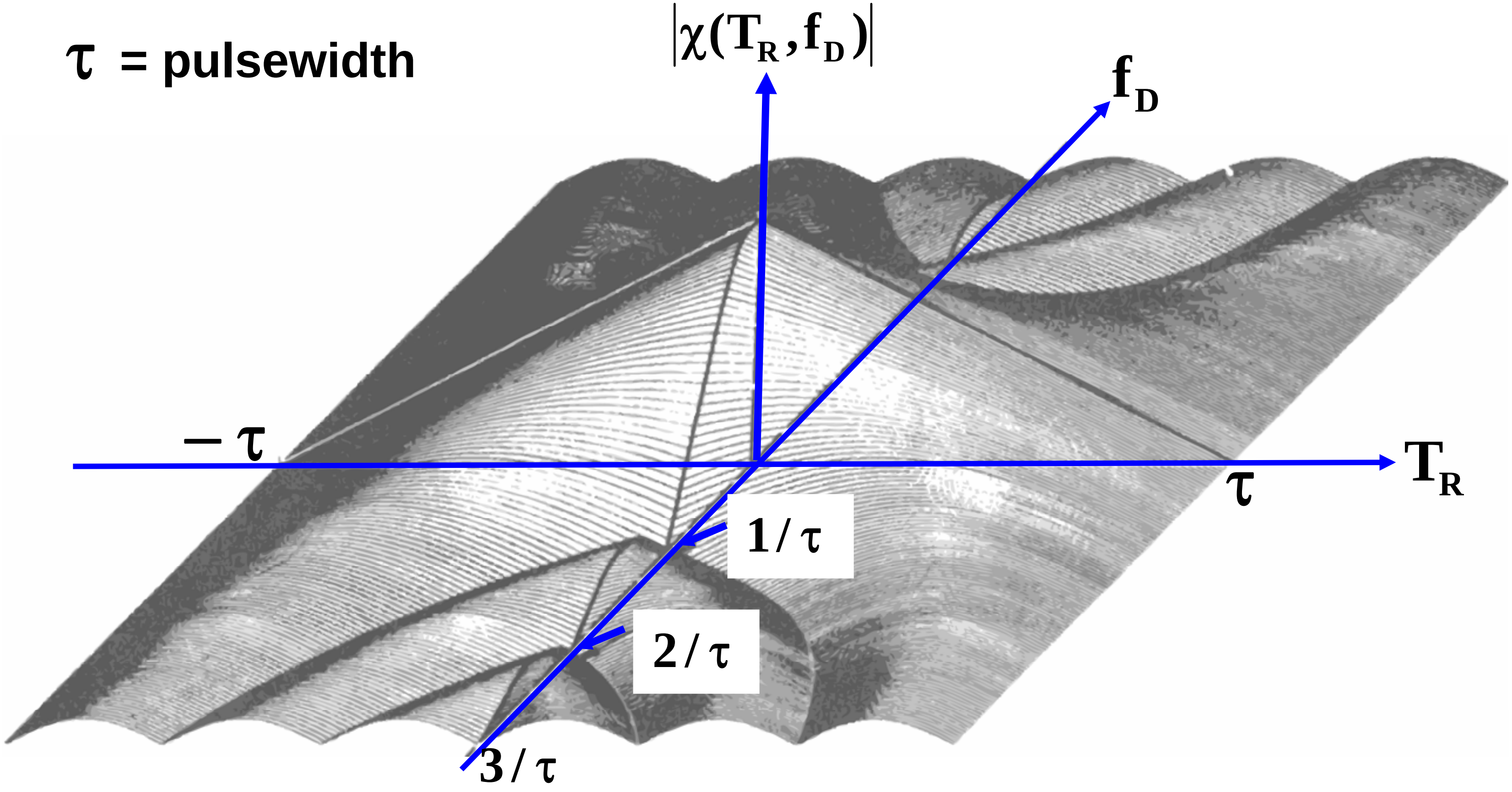
Long pulsewidth

Short pulsewidth

f_D = Doppler frequency shift T_R= Time delay τ = pulsewidth

Ambiguity Function of Rectangular Pulse

τ = pulsewidth

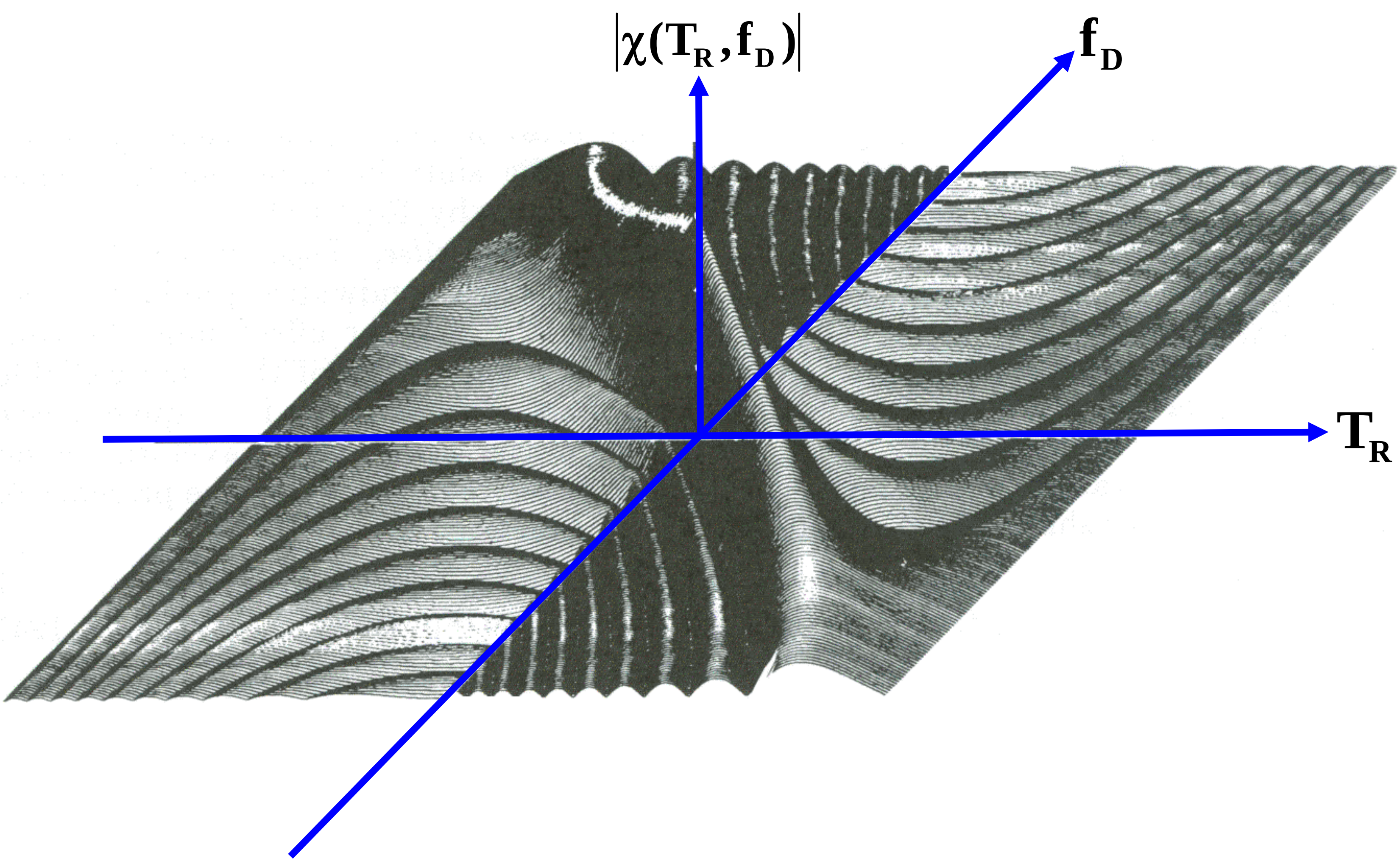


Triangular shape along time axis

$(\sin x)/x$ shape along frequency axis

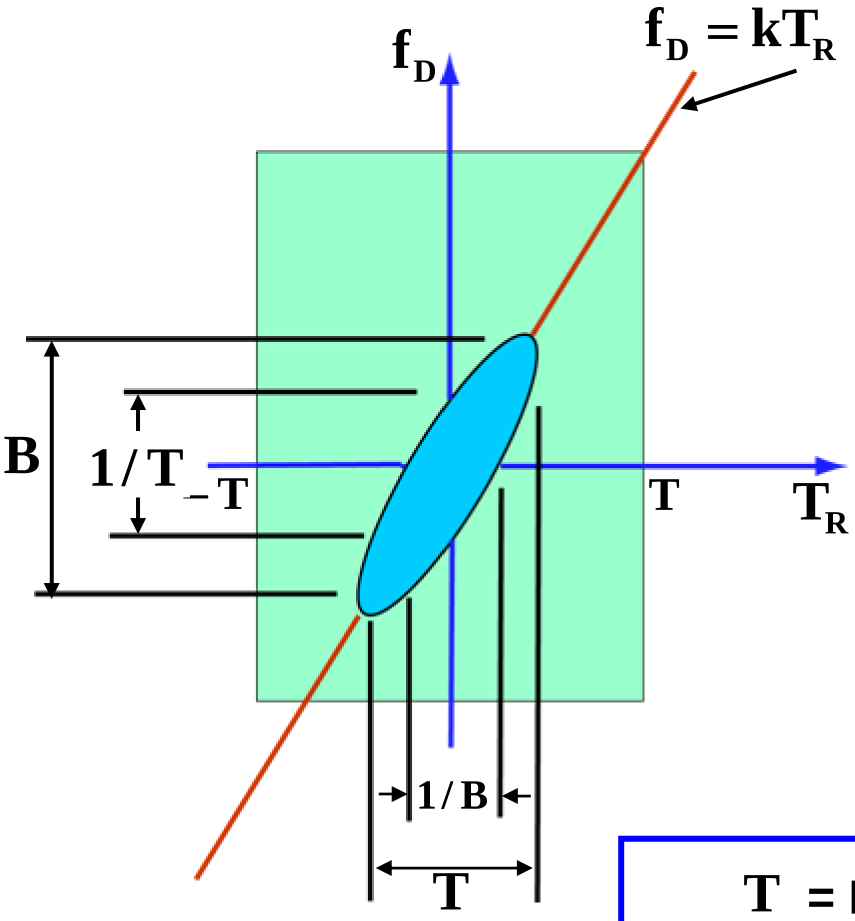
Adapted from Rihaczek, in Skolnik, Reference 13

Ambiguity Function of Linear FM Pulse



Adapted from Rihaczek, in Skolnik, Reference 13

Ambiguity Function of Linear FM Pulse



- **Ridge (knife edge) in Ambiguity diagram illustrates range Doppler coupling in linear FM waveform**
- **In this case, $BT >> 1$**
- **Angle of ridge is determined by the slope B/T**

T = Pulsewidth

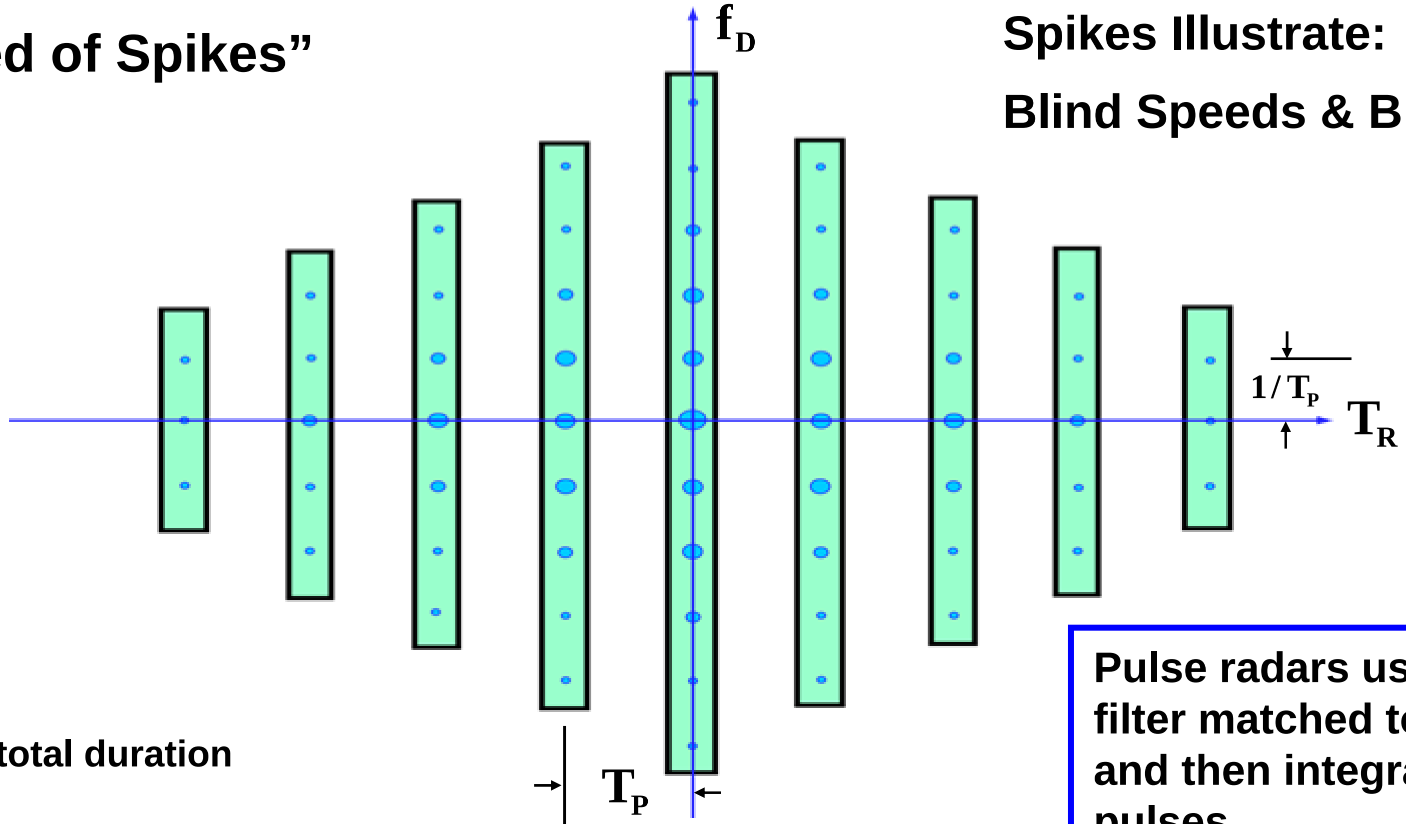
B = Bandwidth = $f_2 - f_1$

Ambiguity Function for a Burst of Five Rectangular Pulses

“Bed of Spikes”

Spikes Illustrate:

Blind Speeds & Blind ranges

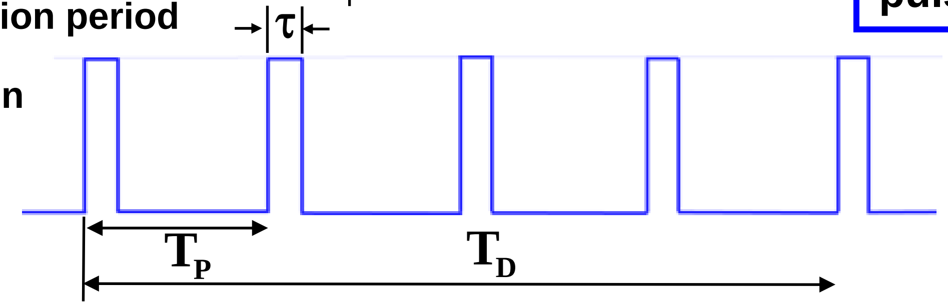


Pulse radars usually use a filter matched to one pulse and then integrate the N pulses

T_D = total duration

T_P = pulse repetition period

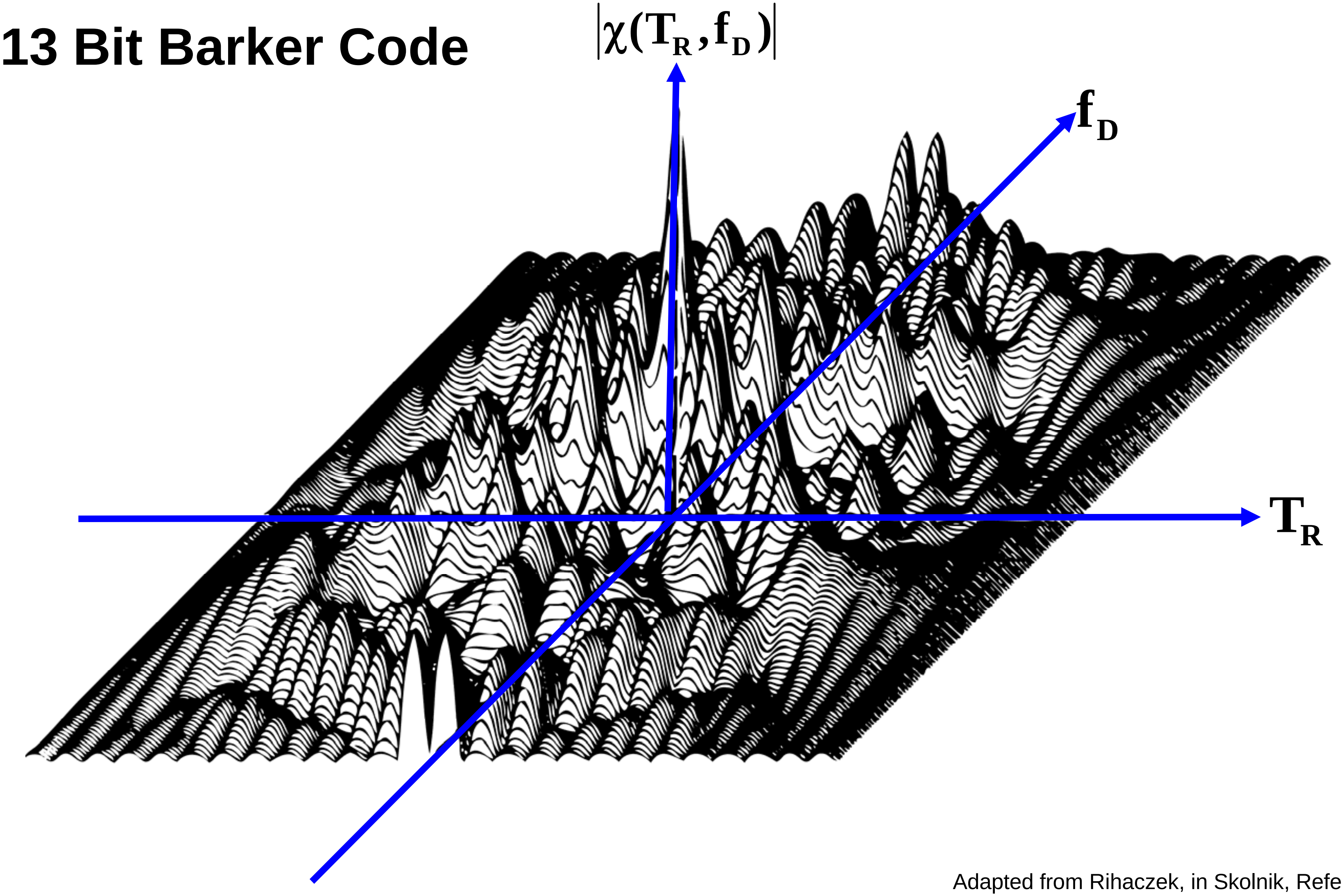
τ = pulse duration



5 pulse burst

Ambiguity Diagram for Phase Coded Pulse

13 Bit Barker Code



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  - ➡ – Preview of Airborne Pulse Doppler issues

- Summary

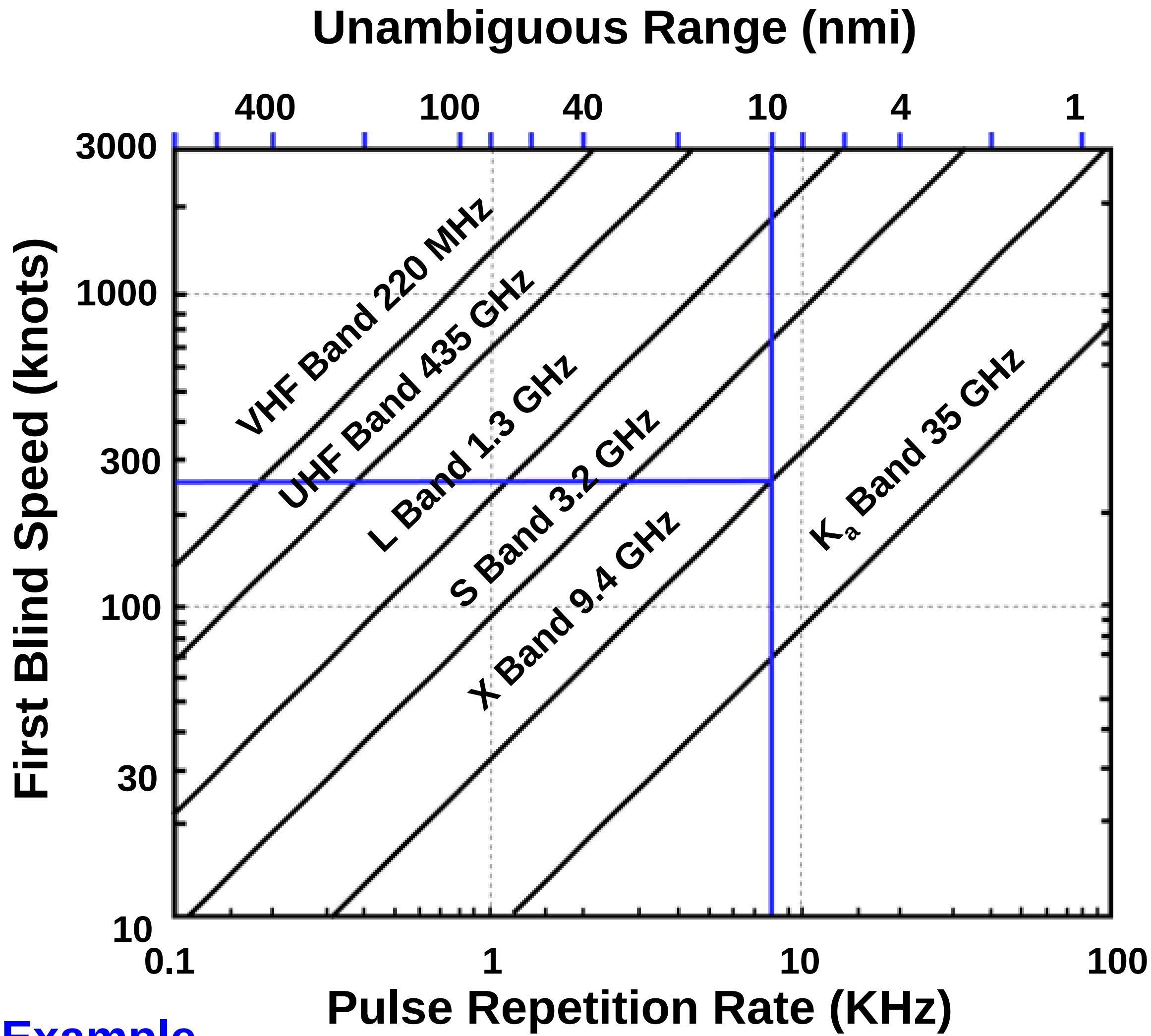
Pulse Doppler Radar Techniques on Airborne Platforms

Courtesy of US Air Force

Courtesy of US Navy

Courtesy of US Air Force

Doppler Velocity - Range Ambiguity Issues



Combining

$$V_B = \frac{\lambda f_{PRF}}{2}$$

and

$$R_U = \frac{c}{2 f_{PRF}}$$

Yields

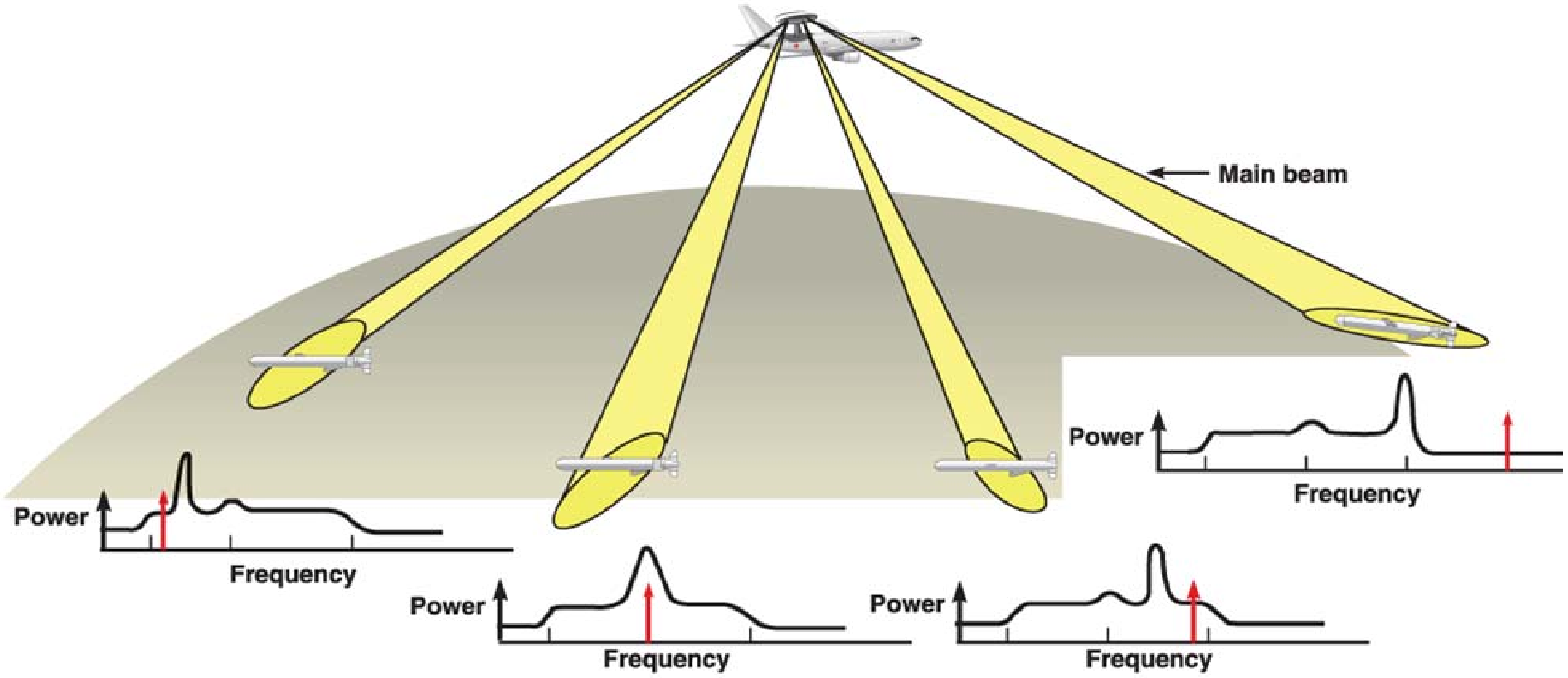
$$V_B = \frac{\lambda c}{4 R_U}$$

Example

X-Band Fighter Radar ➡ $R_U = 10\,\text{nmi} \; - f_{PRF} \sim 8\text{ KHz} - V_B \sim 270\text{ knots}$

Airborne Radar Clutter Characteristics

Illustrative example **Without Pulse-Doppler ambiguities**



- **Doppler frequency of mainbeam clutter depends on scan direction**
- **Doppler frequency of target depends on scan direction and aspect angle**

Viewgraph Courtesy of MIT Lincoln Laboratory
Used with permission

Summary

- **Pulse Doppler techniques can be used to optimally reject various forms of radar clutter**

- **Moving Target Detector is an example of near-optimum Doppler processing and associated adaptive thresholding techniques implemented in low PRF radars**

- **Ambiguities in range and Doppler velocity can be resolved by transmitting multiple bursts of pulses with different PRFs**
  - **The Chinese remainder Theorem is a useful tool in resolving these ambiguities**

- **The ambiguity function is a useful tool to understand the time and frequency properties of different waveforms**

Homework Problems

- From Skolnik (Reference 1)
  - Problems 3-9, 3-10, 3-11, 3-12, 3-13, 3-14 and 3-15

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